COMPUTER AIDED LINE INTEGRAL CALCULUS IN ENGINEERING

Matilde Legua[1], *José A. Moraño*[2] *and Luis M. Sánchez Ruiz*[3]

***Abstract* ¾** *Spanish Engineering Schools are evolving by changing their study programs. In the last years at the Escuela Universitaria de Ingeniería Técnica Industrial de Valencia (EUITI Valencia, Spain) we have been trying to harmonize the learning of mathematics courses as engineering tools with the common use of mathematical software in order to solve engineering mathematical problems. The aim of this paper is to focus on line integrals which do have many engineering applications and survey different techniques which enable to take advantage of the mathematical software available to engineering students, so that it can be used in the classroom.*

Index Terms ¾ Engineering Education, Mathematics integration in Engineering courses, Line integral, DERIVE, MATHEMATICA, MATLAB.

ENGINEERING NEEDS

If we had to define the present situation of the world with just one word, quite likely we would choose changing, which in fact is not far away from challenging. And so the needs of future Engineering professionals do not escape of the general situation, their needs are changing and adaptability and constant update is nowadays more important than ever and this tendency will increase with no doubt in the future.

We have heard from our Rector Justo Nieto that he would be happy if he might design some Engineering studies in which he were allowed to leave the last semester in blank to be filled within 4 years, when the students were to follow it. We do think that he is right in the sense that we must be aware that a lot of our teaching will become obsolete in the next future and we cannot know at this moment what the needs of our students will be when they graduate.

SPANISH BACKGROUND

The Ley de Reforma Universitaria (LRU in the sequel) is the present inforced Spanish law concerning the university structure which dates back to the early eighties when it was approved by the Central Government enabling the development of different syllabuses throughout Spanish universities in order to prepare them to face the 21st Century from a more advanced and flexible framework.

As a consequence of it, engineering education moved into a credit system, with the theoretical contents of many subjects greatly reduced, increasing the student's capability to choose and design part of his curricula. This meant reviewing the contents of all the subjects and changing the way of teaching if just compressing previous teaching programmes was to be avoided.

However the solutions found in the different Spanish universities is far away from being uniform. Here we expose the general orientation given at the Escuela Universitaria de Ingeniería Técnica Industrial, EUITI, de Valencia (SPAIN) and explain how we have focussed it in Mathematics subjects. We will particularize in a special topic as is the line integrals calculus, one of the most usefuls tools in Engineering.

EUITI BET

Since the late eighties, EUITI has favoured a model of teaching which encourages the student to develop a series of attitudes within the pedagogical process in such a way that they are strongly interested in their own training.

EUITI has laid its foundations for this aim rooting its model on three main bets:

- Methodology innovation programmes and efficient use of multimedia technology used as a training tool.
- Involvement in international collaborative and exchange programmes.
- Relationship with industry.

This paper can find somehow its origin in the first of the above. And in fact they were a way of waiving the reduction of teaching hours within the new University curricula which required a greater balance between theory and practice. And what at first sight some mathematicians might see as a waste of time by taking part of their classes to the use of computers instead of delivering blackboard classes, has later proved to be an efficient way of not wasting time in routine calculations, so getting to recover some of the time lost for theoretical expositions with the arrival of the LRU.

On the other hand, let us mention that Collaborative Programs of EUITI: ERASMUS, SOCRATES, LINGUA, LEONARDO, TEMPUS, ALFA and with US institutions have clearly helped to develop and update our subjects, on occasions just to avoid lack of knowledge of some specific issue by our students that were to spend part of their educational period in some partner institution.

Luckily scientific software is a subject that has passed with Honours the Globalization test currently spread out and

[1] Matilde Legua, Centro Politécnico Superior, María de Luna 3, Universidad de Zaragoza, E-50015 Zaragoza (Spain), mlegua@posta.unizar.es
[2] José Antonio Moraño, EUITI, Universidad Politécnica de Valencia, E-46022 Valencia (Spain), jomofer@mat.upv.es
[3] Luis Manuel Sánchez Ruiz, EUITI, Universidad Politécnica de Valencia, E-46022 Valencia (Spain), lmsr@mat.upv.es

is well ahead of other subjects. Thus any effort done to understand and use any technological resource related to software is highly rewarded when its use is required far from your home country, a fact that does not happen with other subjects: rules, regulations and laws may differ from one place to another.

Accordingly, nowadays the use of computer classrooms is standard at EUITI, not just spending some time making laboratory work, but a powerful tool commonly used and available at any moment.

Our computer classrooms are shared with other subjects: Physics, Computer Fundamentals, Technical Drawing and Languages. In exchange we do not have to worry for its maintenance and we count with technicians taking care of the hardware and even the software. Teachers just have to worry for the mathematical content of the sofware used by the students.

And if our way of teaching has evolved, so have the exams that the students must pass. Up to the moment we have not dared to abandon completely classical exams with theoretical/practical requirements, but we have also incorporated the requirement of compulsorily passing exams done with computers.

COMPUTER AIDED LINE INTEGRAL CALCULUS

And when at the end the decision of incorporating some mathematical software into the classroom is taken, comes a difficult question: Which package to choose?

As almost everything in life and teaching is not an exception, the answer is not simple. It depends on what we aim to do, the time we can spend on and of course, thought it should not influence, the preferences of the teacher.

As previously mentioned, in this paper we are going to address a subject commonly studied at Engineering courses, the line integral one of which most important physical applications is that of finding the work W done on a particle moving along a curve $c(t)$ in a force field F, namely

$$W = \oint_c F dr \ .$$

When $c : [a,b] \rightarrow \Re^3$ is a smooth curve, the line integral can be calculated by means of the expression

$$W = \oint_c F dr = \int_a^b F\left(c(t)\right) \cdot c'(t) dt$$

which calculates the work done to move the particle from $c(a)$ to $c(b)$.

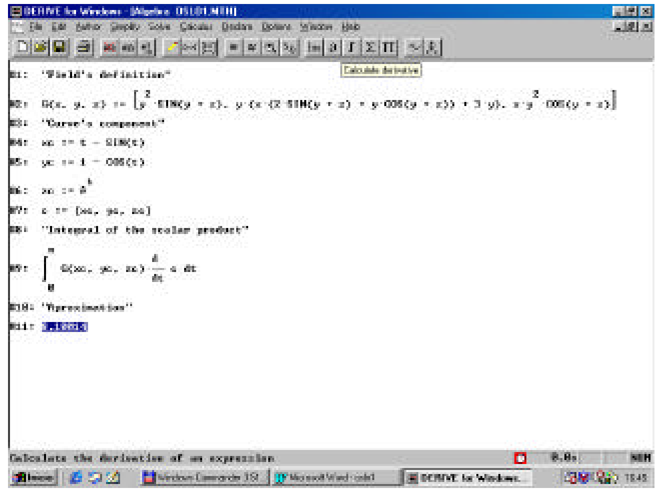
Let us see how some of the most commonly used Mathematical software: DERIVE, MATHEMATICA and MATLAB deal with this problem in a particular example. For instance, let us assume that

$$c(t) = (t - \sin t)i + (1 - \cos t)j + e^t k, \quad t \in [0, p]$$

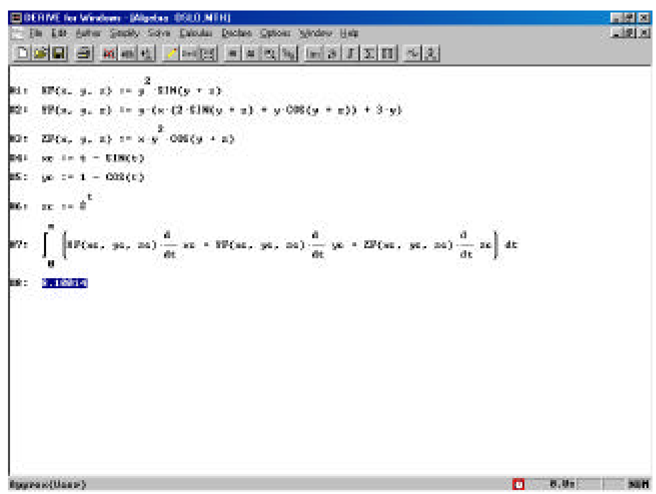
is the trajectory path that the particle follows, and that the force field F has $y^2 sin(y+z)$ as i-component, $y(x(2sin(y+z)+ycos(y+z))+3y)$ as j-component and $xy^2 cos(y+z)$ as k-component.

There are several ways of using the Mathematical software to evaluate the corresponding line integral. We start showing the possibilities of DERIVE.

One of the most simple ones is just to introduce the data in the program and ask it to follow the steps we would follow without computer assistance.

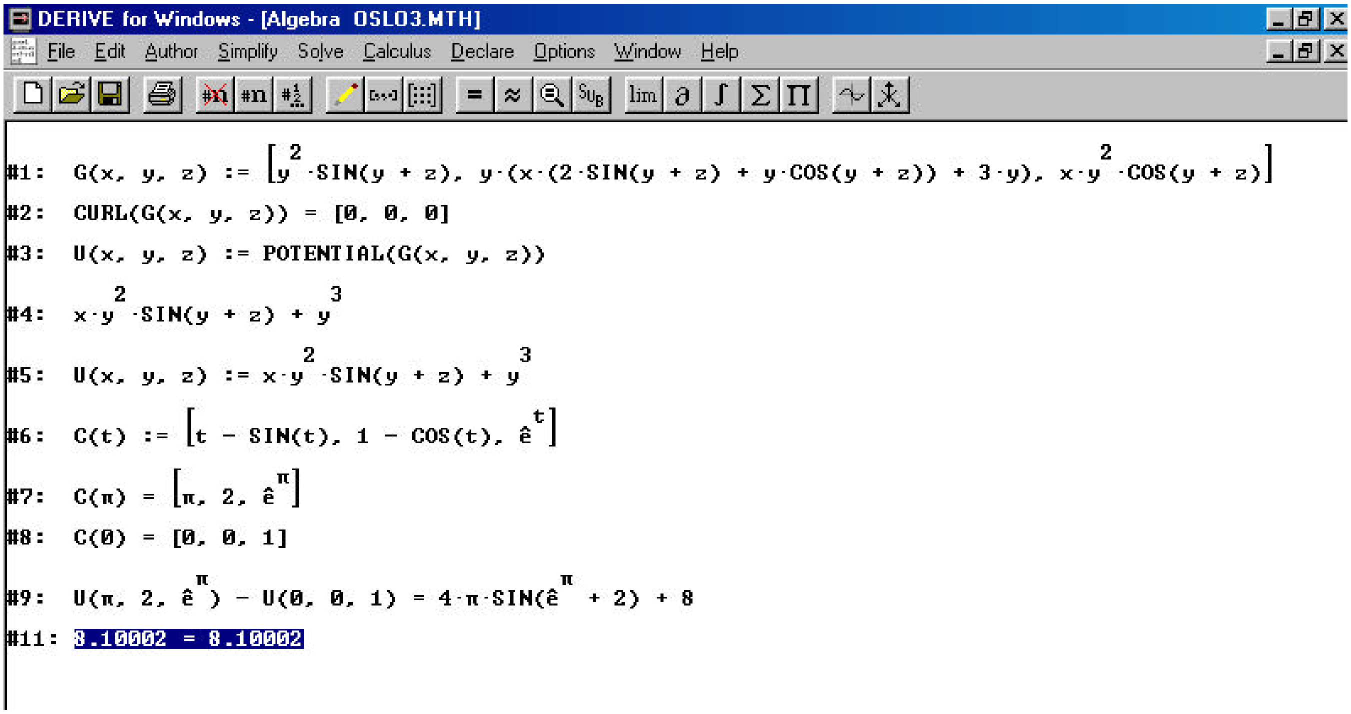


Another way to do the above calculation is by defining separately the field components and do the scalar product later.

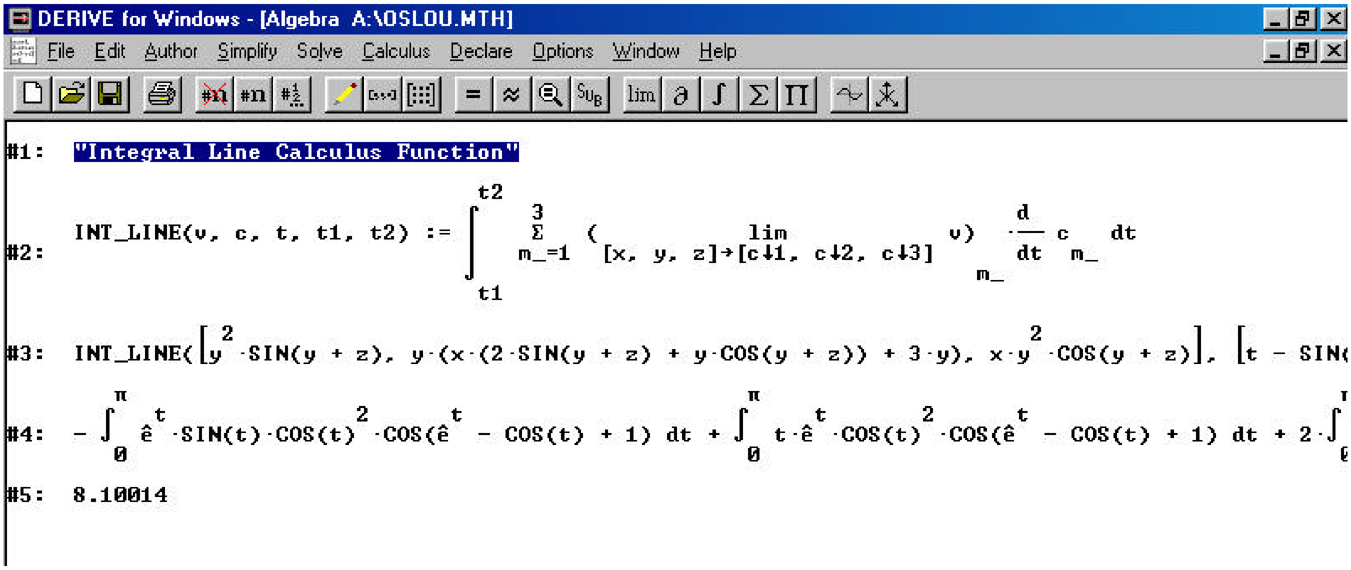


In the particular case in which the curve c is piecewise smooth and is located in an open connected region R and the vector field is conservative in R, then the line integral between any two points A and B is just the difference of the antiderivative function evaluated in B and A, the line integral being independent on the integration contour.

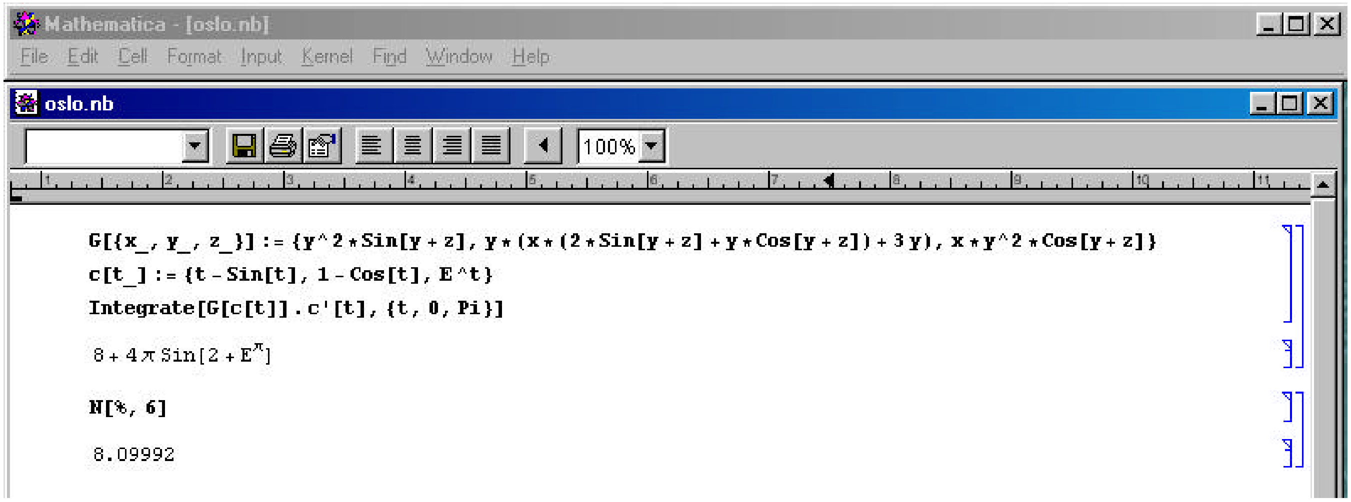
In order to prove that the vector field is conservative we have to check whether its curl is zero and, if affirmative, then calculate the antiderivative function of the vector field.



Finally we are also able to calculate the line integral by defining a function with five arguments: the vector field, the path, the parameter t and its limits a and b. If we do this and keep it in a separate file for ulterior use, line integral calculus becomes really easy with DERIVE.



In what concerns another commonly used program, MATHEMATICA, it also enables to calculate the above by following the general procedure to evaluate line integrals and properly using the corresponding commands.

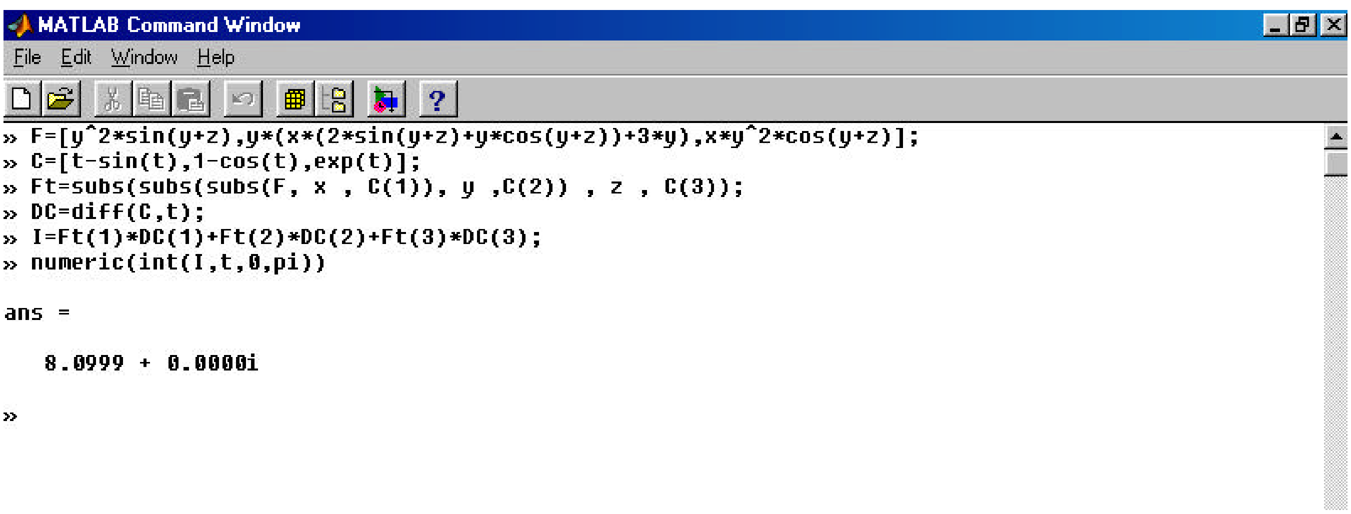


MATLAB is well known as a powerful numeric program, and very well fit for calculations involving matrices.

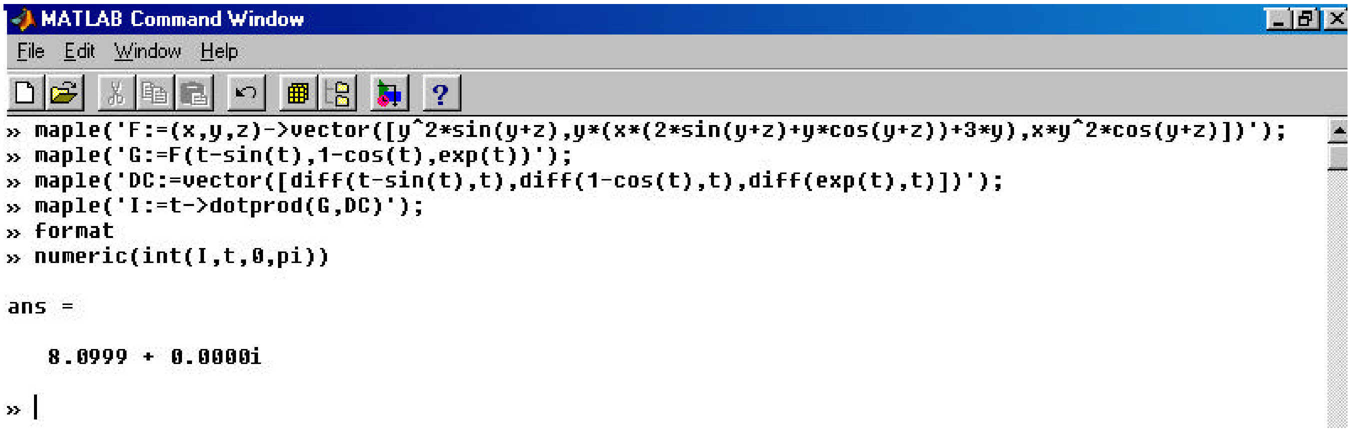
However it can handle symbolic mathematical calculations if we declare them that way by means of the *syms* command.

As usual intermediate MATLAB calculations may be omitted by writing ';' at the end of each command.

We are going to expose three different ways to calculate this particular line integral. The first of them consists of making MATLAB to follow each required step as we have already done with DERIVE and MATHEMATICA.

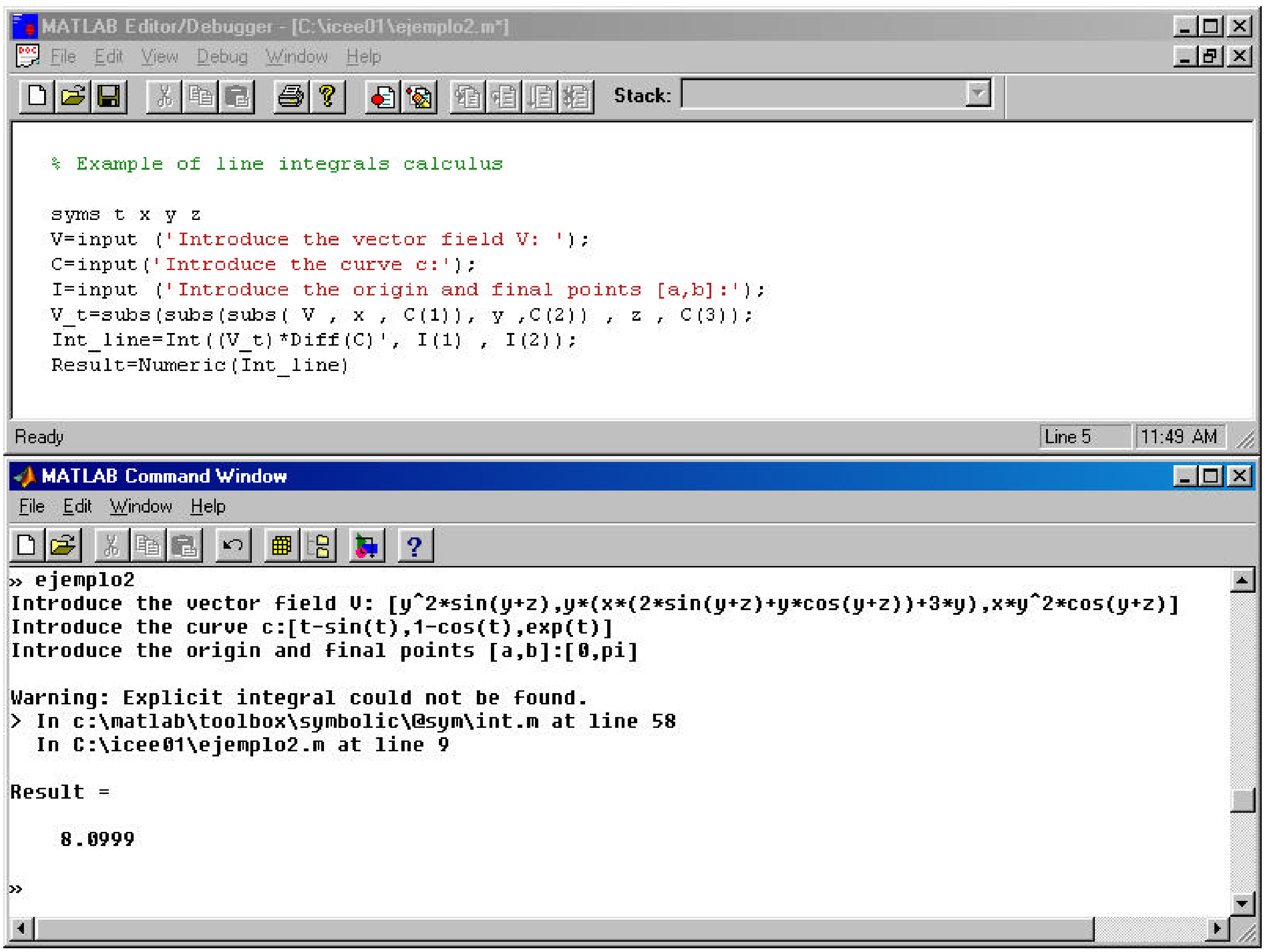


MATLAB can also use MAPLE V libraries to do symbolic calculations. This is only possible if the "Extended Symbolic Math Toolbox" has been installed. Lat us recall that this is done by writing *maple* before the corresponding needed function:



Finally, let us recall that we can take advantage of MATLAB capabilities to create an M-file for ulterior use.

The next image include the M-file we have created for this purpose and how we have used it.



CONCLUSSION

After the comparison of methods, DERIVE seems to be the most efficient program in order to evaluate line integrals for several reasons, mainly its easiness to be implemented and syntax much easier to learn and use compared to MATHEMATICA and MATLAB. In fact, once implemented its use is completely analogue to other functions implemented in the INT_APPS.MTH file of the program.

Mathematics teaching is facilitated with the use of computers. But definitely a little of the specific capabilities and way of working of the software used by the student must be known by him in order to take real advantage of it, and this 'little' is less with DERIVE than with the other compared programs.

This particular issue we have addressed: computer aided line integral calculus, gives a chance to recall the student how to calculate them by hand and to introduce him in developing their own tools with the program when needed in such a way that they can personalize their DERIVE to match their needs, something that undoubtedly they will have to do with any scientific software as long as their needs go beyond the standard and basic ones.

REFERENCES

1. Ballester, E, Sánchez Ruiz, L.M. and Seiz, R. "Teaching Innovation through Collaborative Programs at EUITI Valencia (Spain)", *CD International Conference on Engineering Education*, Oslo 2001, Paper no. 174.
2. Legua, M., Moraño, J.A. and Sánchez Ruiz, L.M., "An Iterative Calculus Programming Application", *CD International Conference on Engineering Education,* Taipei 2000 , Paper no. WC1-2.
3. Legua Fernández, M., Sánchez Ruiz, L.M. and Morales González, I., "Matemáticas para Ingenieros Asistidas por Ordenador", *Servicio de Publicaciones de la Universidad Politécnica de Valencia* , 1998
4. Pérez, C. "Análisis matemático y Álgebra lineal con Matlab ", *Ra-ma*, 1999.
5. Rodríguez Gómez, F.J. and García Mellado, F., "Fundamentos y Aplicaciones de MATHEMATICA", *Paraninfo* 1998
6. Soft Warehouse, "User Manual DERIVE. A Mathematical Assistant for Your Personal Computer", *Soft Warehouse Inc*., 1991.
7. The Student Edition of MATLAB, Version 5 User's Guide. The MATHWORKS Inc. Prentice Hall 1997.
8. Wolfram, S., " Mathematica. A System for doing Mathematics by Computer", Addison-Wesley Publishing Company, 1988.