STRUCTURE VIBRATIONS: AN ON-LINE INTERACTIVE LEARNING ENVIRONMENT

Erica Macho-Stadler[1], Mª Jesús Elejalde-García [2], Angel Franco-García[3] and Jesús Janariz-Larumbe[4]

Abstract — *The aim of this paper is to present our experience focused on the design of didactic units to teach the oscillatory behavior of simple structures. With the utilization of Java technology we develop and build an on-line interactive learning environment comprised on simulations of oscillators and simple structure vibrations that can be used to support lectures for undergraduate students. At the same time, these units are an answer to the number of request from working engineers to update and refresh their technical basic notions. The concisely packaged units include text, examples and graphics, mathematical content, hands-on proposals and questions that accompany the animations.*

Index Terms ¾ Education, Oscillation, Simple Structures, Multimedia, Teaching Materials.

INTRODUCTION

Engineering education is facing a great challenge to meet new demands of the knowledge market as result of the effect of globalization. During the past years there has been an increasing demand for distance learning and on-line learning courses to fulfill both initial and continuing education.

Likewise, in the field of engineering design, vibration analysis is an important factor, since certain resonance may lead to the failure of structures or to noise/sound production. Conventional methods for vibration analysis are based either on theory or on experiments. However experiments can be extremely expensive and practical problems are either too difficult or simply impossible to accomplish by analytical methods. Therefore, numerical simulations play a more and more important role in modern vibration analysis.

Beams and plates (rectangular or circular) with different boundary conditions are key components in mechanical engineering and industrial design. As a matter of fact, we can cite, for example, the components of major structures as bridges, buildings or aircraft wings, circuit boards in electronic industry or some parts of musical instruments.

From an educational point of view, although vibrations of structures can be difficult to understand, the animation of analytical results seems to be a useful tool to improve student's understanding.

BACKGROUND

In a previous work [1], the authors have proposed a project focused towards the development of an education module of Acoustics. The module was presented in the form of an interrelated set of didactic materials: on-line materials [2] and books and CD-ROM materials.

On-line materials are thought to be a module of Acoustics for engineering students or engineers' continuing formation developed in various chapters. In their design we take into account several features such as gradual progress, completion, flexibility and easy accessibility. The books and CD-ROM are specifically designed as a helping and complementary tool for teachers.

The complete module will cover some aspects of Acoustics, focused around Basic Principles, Musical Acoustics and Noise. The list of these general topics is: Basic Principles of Acoustics: Vibrations and Waves; Musical Acoustics: Musical Instruments; Architectural Acoustics; Electroacoustics; Noise and Physiological Acoustics.

On-line materials include text, examples and graphics, mathematical content, hands-on proposals and questions that accompany the animations. Their general schema is:

- Objective
- Description
- Examples and Simulations
- Questions
- Bibliographic, Multimedia and Web resources
- Self exam
- Links

STRUCTURE VIBRATIONS

The aim of this work is to present some aspects of the didactic units that we are developing in order to teach the oscillatory behavior of simple structures. These units are included in the general topic "Basic Principles of Acoustics: Vibrations and Waves", but they are related with others as "Musical Acoustics: Musical Instruments" or "Noise".

The vibration properties of some one-dimensional (strings and bars) or two-dimensional continuous systems

[1] Erica Macho-Stadler, Universidad del País Vasco, Escuela Superior de Ingenieros, Alameda de Urquijo S/N, 48013 Bilbao (Spain), wupmaste@bi.ehu.es
[2] Mª Jesús Elejalde-García, Universidad del País Vasco, Escuela Superior de Ingenieros, Alameda de Urquijo S/N, 48013 Bilbao (Spain), wupelgam@bi.ehu.es
[3] Angel Franco-García, Universidad del País Vasco, Escuela Universitaria de Ingeniería Técnica Industrial, Avenida Otaola 29, 20600 Eibar (Spain), wupfrgaa@sc.ehu.es
[4] Jesús Janariz-Larumbe, Universidad del País Vasco, Escuela Superior de Ingenieros, Alameda de Urquijo S/N, 48013 Bilbao (Spain), wupjalaj@bi.ehu.es

(membranes and plates) are significant in later understanding and knowledge of the dynamic analysis of structures. We study how motions of strings, bars, membranes and plates can be described, the influence of boundary conditions and how the vibration frequency dependence on some physical parameters is [3-6].

In order to achieve the best understanding of this vibration behavior, we combine lectures and lab experiments. We try to stimulate the creative and motivated self-study by means of the study guide that also enables to check the actual knowledge through selected self-exam questions. In each case, the guide summarizes the objectives and the description of the phenomena and it also introduces the lab experiments. On the one hand, we consider Hands-on Lab experiments that allow students to visualize specific phenomena and obtain experimental skills [7]. On the other hand, Virtual Lab experiments based on Java applets allow the manipulation of parameters, which are difficult to control in real lab experiments, and visualization of theoretical processes, in an interactive environment. These experiments can be performed at home, through the Internet. In addition, teacher can introduce them in a computer room as Prelab exercises, before starting the conventional lab experiments. In both cases, it is important to follow the step-by-step proposed methodological sequence avoiding the compulsive "click" touch.

In terms of multiple representations, the main goal of solving physics problems is to represent physical processes in different ways (words, sketches, diagrams, graphs, and equations) rather than relying on formula-centered techniques that lack qualitative understanding. The abstract verbal description is linked to the abstract mathematical representation by the more intuitive pictorial and diagrammatic physical representations. The qualitative physical representations build a bridge between the verbal and the mathematical representations, helping students to move in smaller and easier steps from words to equations. They also help students to develop images that give the mathematical symbols meaning. After representing the process, students can obtain a quantitative answer to the problem using the mathematical representation.

In the next sections we include the objectives, descriptions and some simple examples and simulations of the vibration behavior of strings, bars, membranes and plates. Besides, there is a general description of the applets construction.

TRANSVERSE VIBRATIONS OF STRINGS

Objective

We shall study the transverse motion of the string. We shall focus on the fundamental resonance frequency f_1, the frequency of successive overtones and their relationship with f_1.

Description

We shall use the string of length L under a tension T. We assume that the string has negligible transverse dimension, that its mass is distributed uniformly (e per unit length), that is perfectly flexible and that it is connected to massive non-yielding supports.

If we suppose a string stretched under a tension T, its equilibrium position will be straight-line. We designate a point on the string by giving its distance x from some origin while in its equilibrium position. The motions we shall study involve the transverse displacement of the point x an amount y to the equilibrium line. Transverse waves are propagated along the string with a velocity c. The relationship between the dependence of y on x and its dependence on time t is given by the second-order equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} = \frac{T}{e}\frac{\partial^2 y}{\partial x^2} \qquad (1)$$

which is called the wave equation.

Actual strings are fastened at both ends, so that two boundary conditions are imposed: y must always be zero both at $x = 0$ and at $x = L$. The general solution represents two waves of the same frequency and wavelength, traveling in opposite directions along the strings. If the amplitudes of the two simple-harmonic waves (the normal modes of vibrations) are equal the combination is called a standing wave. The points where the two traveling waves always cancel each other and the string never moves are called the nodal points of the wave motion. Halfway between each pair of nodal points is the part of the string having the largest amplitude of motion, where the two traveling waves always add their effects. This portion is called a loop or antinode.

The different allowed simple-harmonic motions are all given by the expression

$$y = A_n sin\left(\frac{pnx}{L}\right)cos\left(\frac{pnc}{L}t - F_n\right)$$

$$n = 1,2,3,4,... \quad f_n = \frac{nc}{2L} = \frac{n}{2L}\sqrt{\frac{T}{e}} \qquad (2)$$

The lowest allowed frequency is called the fundamental frequency. The higher frequencies or overtones are integral multiples of the fundamental frequencies. Overtones satisfying this simple relation to the fundamental are called harmonics. The fundamental frequency is called the first harmonic, the first overtone is called the second harmonic, and so on.

Examples and Simulations

Very few vibrating systems have harmonic overtones, but these are the basis of nearly all-musical instruments. If the overtones are harmonics, the sound seems particularly pleasant to the ear.

Figure 1 shows the applet that represents the standing waves in a string under tension It allows the observation of the vibration modes by pushing the Start, Pause, Continue, Step and Scale buttons. We can control the Frequency and the Velocity of the wave.

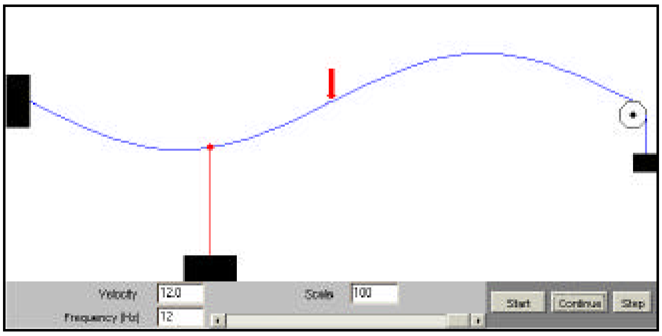

FIGURE. 1
STANDING WAVES IN A STRING UNDER TENSION.

LONGITUDINAL VIBRATIONS OF BARS

Objective

We shall study the longitudinal motion of a bar. We shall focus on the fundamental resonance frequency f_1, the frequency of successive overtones and their relationship with f_1.

Description

Longitudinal waves in a thin bar travel at the same velocity, as do longitudinal waves in a string of the same material. Suppose we have a long, thin bar under no tension. We assume that the bar is straight, with uniform cross section and symmetrical about a central plane. The length of the bar is L, its density is r, and its Young's modulus is E. We designate a point on the bar by giving its distance x from some origin while in its equilibrium position. The motions we shall study involve the axial displacement w of a small volume element. Longitudinal waves are propagated along the bar with a velocity c_L. The relationship between the dependence of w on x and its dependence on time t is given by the second-order equation:

$$\frac{\partial^2 w}{\partial t^2} = c_L^2 \frac{\partial^2 w}{\partial x^2} = \frac{E}{r}\frac{\partial^2 w}{\partial x^2} \qquad (3)$$

Different boundary conditions give different solutions for this equation. When both ends are fixed, or if both are free, the different allowed simple-harmonic motions are all given by the expression:

$$w_n = A_n sin\left(\frac{pnx}{L}\right) cos\left(\frac{pnc_L}{L}t - F_n\right)$$

$$n = 1,2,3,4,... \quad f_n = \frac{nc_L}{2L} = \frac{n}{2L}\sqrt{\frac{E}{r}} \qquad (4)$$

Longitudinal waves in a thin bar are nondispersive, i.e. their speed c_L does not depend on frequency. In a thick bar, the longitudinal wave speed decreases slightly at high frequency due to the effect of lateral inertia.

Examples and Simulations

Rods in which the longitudinal waves are excited by striking the ends are used as standards of high frequency sounds, bigger than 5000 Hz, where a tuning fork is not very satisfactory.

Figure 2 corresponds to the applet that represents the propagation of a longitudinal wave along an elastic bar. Likewise, it shows the characteristics of the harmonic wave motion. In the origin there is a source that describes a simple harmonic motion. The applet allows the observation of the characteristics of the harmonic motion by pushing the Start, Pause, Continue and Step buttons. We can control the Wavelength and the Velocity of the wave.

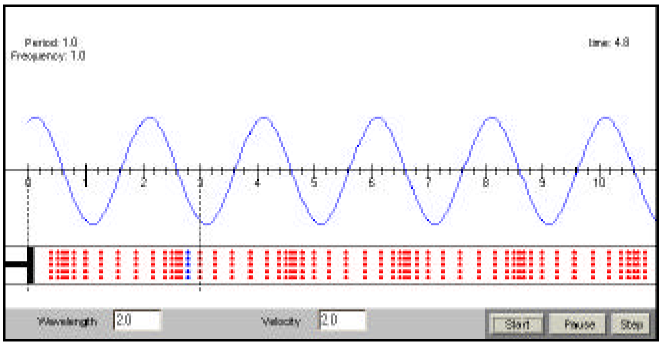

FIGURE. 2
LONGITUDINAL VIBRATIONS OF BARS.

TORSION VIBRATIONS OF BARS

Objective

We shall study the torsion motion of a thin bar. We shall focus on the fundamental resonance frequency f_1, the frequency of successive overtones and their relationship with f_1.

Description

Suppose we have a long, thin bar under no tension. We assume that the bar is straight, with uniform cross section and symmetrical about a central plane. The length of the bar is L, its density is r, and its shear modulus is G. We designate a point on the bar by giving its distance x from some origin while in its equilibrium position. The motions we shall study involve the angle of twist q of a small volume element. Torsion waves are propagated along the bar with a velocity c_T. The resulting wave equation is very similar to the obtained for longitudinal waves. The relationship between the dependence of q on x and its dependence on time t is given by the second-order equation:

$$\frac{\partial^2 q}{\partial t^2} = c_T^2 \frac{\partial^2 q}{\partial x^2} = \frac{G}{r}\frac{\partial^2 q}{\partial x^2} \qquad (5)$$

Different boundary conditions give different solutions for this equation. When both ends are fixed, or if both are free, the different allowed simple-harmonic motions are all given by the expression:

$$q_n = A_n sin\left(\frac{pnx}{L}\right) cos\left(\frac{pnc_T}{L} t - F_n\right)$$

$$n = 1,2,3,4,... \quad f_n = \frac{nc_T}{2L} = \frac{n}{2L}\sqrt{\frac{G}{r}} \qquad (6)$$

Torsion waves in a bar are nondispersive. If the bar is not thin, the torsion wave speed depends on the section of the bar: c_T in a bar of rectangular cross section is less than that of one of circular section.

Examples and Simulations

Bowing a violin string excites torsion waves as well as transverse waves. Torsion waves affect the mechanism of the bow/string interaction.

For the moment we have not designed any simulation of torsion vibrations in bars.

BENDING VIBRATIONS OF BARS

Objective

We shall consider transverse motion of the bar due to the bending moment only (the motion of the bar is supposed to be perpendicular to its central surface). We shall focus on the different allowed motions, the fundamental resonance frequency f_1, the frequency of successive overtones and their relationship with f_1.

Description

We shall study a long, thin bar or rod under no tension. We assume that the bar is straight, with uniform cross section and symmetrical about a central plane. The length of the bar is L, its density is r, its Young's modulus is E and the radius of gyration of the cross section is k.

When the bar is bent, its lower half is compressed and its upper half stretched (or vice versa). The equation of motion of the bar is

$$\frac{\partial^4 y}{\partial x^4} = -\frac{r}{Ek^2}\frac{\partial^2 y}{\partial t^2} \qquad (7)$$

This is a fourth-order differential equation. It is not possible to construct a general solution from transverse waves traveling along the bar with constant velocity and unchanged shape. The velocity of transverse waves is, in fact, quite dependent on frequency; that is, the bar has dispersion. The general solution is

$$y(x,t) = \{ A cosh(kx) + B sinh(kx) + C cos(kx) + D sin(kx) \} e^{-iwt} \qquad (8)$$

where we have defined the propagation number $k = w/c$ and A, B, C, and D are real constants. We need four boundary conditions (two at each end) to determine them. There are three different end conditions for a bar: free, supported (hinged), and clamped. For each of these a pair of boundary conditions can be written. At a free end, there is no bending moment and no shearing force, so the second and third derivatives are zero. At a simply supported end, there is no displacement and no torque, so y and its second derivative are zero. At a clamped end, there is no displacement and no velocity, so y and its first derivative are zero.

If we consider a bar of length L clamped at $x = 0$ and free at $x = L$, we obtain two equations that fix the relationship between A and B and between k and L.

$$B = -A\frac{cos(kL) + cosh(kL)}{sin(kL) + sinh(kL)}$$

$$coth^2(kL/2) = tan^2(kL/2) \qquad (9)$$

The solutions of this transcendental equation in order of increasing value are $k_n L/p = b_n$, where $b_1 = 0.597$, $b_2 = 1.494$, $b_3 = 2.500$, etc. It turns out that b_n is practically equal to $n - \frac{1}{2}$ when n is larger than 2. For the allowed values of the frequency we have

$$f_n = \frac{k_n^2}{2p}\sqrt{\frac{Ek^2}{r}} = \frac{p}{2L^2}\sqrt{\frac{Ek^2}{r}} b_n^2 \qquad (10)$$

The allowed frequencies depend on the inverse square of the length of the bar and the overtones are not harmonics. Other end conditions combinations can be obtained in a similar way.

Examples and Simulations

A tuning fork can be considered to be two vibrating bars, both clamped at their lower ends. If the bars are struck, the initial metallic "ping" rapidly dyes out, leaving and almost pure tone due to the fundamental.

Figure 3 and Figure 4 correspond to the applet that shows the first five characteristic functions for a vibrating bar clamped at one end and free at the other.

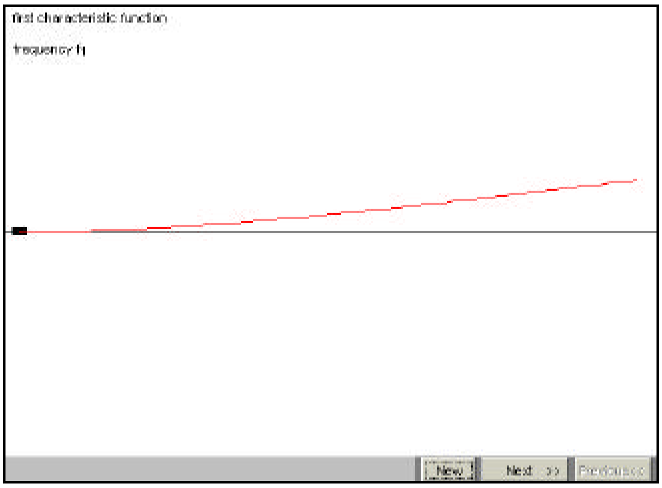

FIGURE. 3
FIRST CHARACTERISCTIC FUNCTION FOR A BAR CLAMPED AT ONE END AND FREE AT THE OTHER.

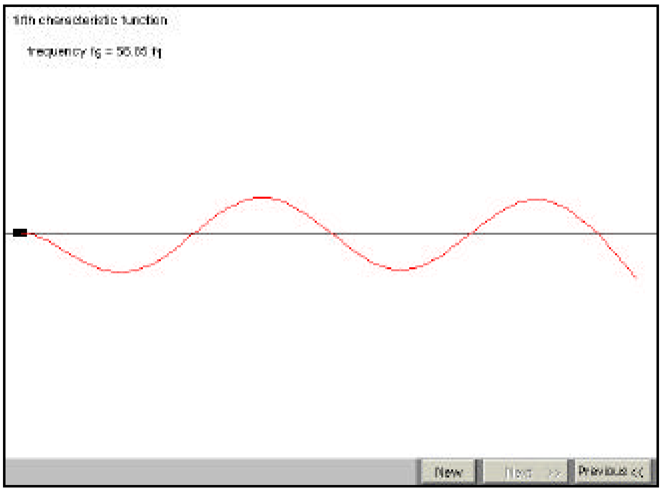

FIGURE. 4
FIFTH CHARACTERISCTIC FUNCTION FOR A BAR CLAMPED AT ONE END AND FREE AT THE OTHER.

Figure 3 shows the first characteristic function and Figure 4 shows the fifth. We interact by pushing the New, Next and Previous buttons. As it can be seen for the higher overtones most of the length of the bar has the sinusoidal shape of the corresponding normal mode of the string, with the nodes displaced toward the free end.

MEMBRANES

Objective

We shall study waves on membranes. We shall focus on a resonance frequency *f*. Also we shall study the standing wave patterns (Chladni patterns).

Description

We shall study a perfectly flexible membrane. We assume that the membrane is pulled evenly around its edge with a tension *T*. The superficial density of the membrane is s. The displacement of the membrane from its equilibrium position is h. The wave equation should be written as:

$$\frac{\partial^2 h}{\partial t^2} = c^2 \nabla^2 h = \frac{T}{s} \nabla^2 h \qquad (11)$$

For a circular membrane and harmonic solutions, using polar coordinates, we find that the possible solutions for simple-harmonic oscillations are

$$h(r,f,t) = AJ_m(kr) \cos mf e^{iwt} \qquad (12)$$

where $J_m(kr)$ are the ordinary Bessel functions. Each Bessel function goes to zero for several values of kr. The n-th zero of $J_m(kr)$ gives the frequency of the (m,n) mode, which has m nodal diameters and n nodal circles (including one at the boundary).

Examples and Simulations

Membranes are present in some percussion musical instruments, as kettledrums, bass drums, snare drums, tambourines and some others. The condenser microphone corresponds approximately to the case of a circular membrane. The diaphragm of the microphone is metallic, and therefore it has stiffness; but it is often so thin and under such great tension that the effects of stiffness can be neglected.

At present, we are designing the simulation for the vibration of a circular membrane pulled evenly around its edge.

BENDING VIBRATIONS IN THIN PLATES

Objective

We shall study bending waves on thin plates. We shall focus on a resonance frequency *f*. Also we shall study the standing wave patterns (Chladni patterns).

Description

A plate may be likened to a two dimensional bar or a membrane with stiffness. Like a bar, it can transmit longitudinal waves, shear waves, torsion waves or bending waves, and it can have three different boundary conditions: free, simply supported or clamped. The radiation of sound for the bending waves is more important than in the other cases.

We shall study a thin plate under no tension. The thickness of the plate is h, its density is r, its Young's modulus is E and its Poisson ratio is n.

The bending of a plate compresses the material on the inside of the bend and stretches it on the outside. But when a material is compressed it tries to spread out in a direction perpendicular to the compressive force. The displacement of the plate from its equilibrium position is z. The equation of motion is:

$$\frac{\partial^2 z}{\partial t^2} + D\nabla^4 z = \frac{\partial^2 z}{\partial t^2} + \frac{Eh^2}{12r(1-n^2)} \nabla^4 z = 0 \qquad (13)$$

For a circular plate and harmonic solutions, using polar coordinates, we find that the possible solutions for simple-harmonic oscillations are

$$z(r,f,t) = [AJ_m(kr) + BI_m(kr)] \cos mf e^{iwt} \qquad (14)$$

where $k^4 = w^2/D$, $J_m(kr)$ are the ordinary Bessel functions and $I_m(kr)$ the hyperbolic Bessel functions.

Bending waves in a circular plate are dispersive; that is, their velocity $c = w/k$ depends upon the frequency. The frequency of a bending wave is proportional to k^2:

$$f = \frac{w}{2p} = 0.0459k^2 h \sqrt{\frac{E}{r(1-n^2)}} \qquad (15)$$

The allowed values of k that correspond to the normal modes of vibration depend on the boundary conditions. They are labeled k_{mn}, where m gives the number of nodal diameters and n the number of nodal circles in the corresponding normal mode.

Examples and Simulations

The diaphragms of ordinary telephone transmitters and receivers are examples of plates.

For the moment, we have not designed any simulation for the vibration of a plate.

APPLETS

We have two possibilities for designing simulations: We can use specialized programs (Computer Algebra Systems) as Mathematica and Maple V [8], or we can utilize a general proposal language.

We have chosen the second possibility, because, normally, individual users have not CAS programs, and we want our teaching stuff to be accessible for all people interested in Acoustics. In order to create applets that simulate physical phenomena, the elected general proposal language has been Java. These kind of interactive programs are automatically executed when their web page is loaded in a navigator. Besides, they are another element of a web page that includes the text, the mathematical expressions and the necessary figures in order to explain the simulated physical phenomena.

Java language is not usual in the scientific field programming. In this field, FORTRAN and C/C++ languages predominate. However, this new language has all the necessary characteristics for programming the different tasks and its executables work in any computer without any change. This last characteristic is not possible with C or FORTRAN [9].

In this paper we present the simulations of the transversal vibration of a clamped-free bar and that of the vibration of a circular membrane pulled evenly around its edge. Both of them have two parts: the calculation of the frequencies of the different vibration modes and the graphical representation of the characteristic functions corresponding to the vibration modes.

In the first part, an abstract base class is created. This class describes the mid point numerical procedure in order to calculate the roots of the transcendental equation in an interval. In the derived class, we define the transcendent function $f(x)=0$, whose roots we want calculate [10].

In the second part, a user interface is created, in order to visualize the characteristic function of the elected mode in an animated form.

The bar vibration mode applets have been developed using Java 1.1 version, compatible with the current navigators. This type of programs will not present any problem to be adapted to the new version Java 2, when the usual navigators will include the corresponding Java Virtual Machine (JVM).

For the circular membrane, at this moment the applet that calculates the frequencies of the different vibration modes has been developed [11]. In order to represent the characteristic functions we will use a 3D graphical library. It will be no necessary to install additional software in user's navigator to watch the applets.

CONCLUSIONS

In order to achieve the best understanding of the vibration behavior of simple structures we are developing didactic units that present multiple representations of the same phenomena, e.g. mathematical, graphical, animated and textual. The qualitative representations help students to develop images that give the mathematical symbols meaning. The multiple representations seem to be advantageous in improving the students' opportunity to ask questions and to construct their own correct answers.

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