A qualitative study of the role of mathematics in engineering capstone design projects

Authors:

Monica E Cardella, University of Washington, Seattle, WA, monicaeh@u.washington.edu
Cynthia J Atman, University of Washington, Seattle, WA, atman@engr.washington.edu

Abstract — *Design is essential to all branches of engineering—mechanical engineers may design a car or an airplane, an industrial engineer may design a supply chain, a civil engineer a bridge, a chemical engineer a process for converting resources into energy. Each of these engineers completed a rigorous mathematical program to earn the engineering degree, and it may be that it is the mathematics that distinguishes these engineers from other designers, but these engineers' use of mathematics is far less obvious than their design behavior. This paper presents a qualitative study that explores engineering students' use of mathematics in one of the first extended, "real-world" design experiences that engineers encounter: the capstone design project. A team of five Industrial Engineering students was observed while they worked on their capstone project. Additionally, four of these Industrial Engineering students and four students from other engineering departments were interviewed about their use of mathematics in their capstone projects. In this paper we present the analysis of this dataset with the intent to characterize engineering students' use of mathematics while engaged in their capstone design projects. We begin by introducing research, terminology and important concepts from the study of mathematics, design and engineering that are relevant to the current study. Next we describe the specific aims and research questions on how engineering students use math in the context of design that frame this study. Following this, we provide a description of the study's design, and the results of an analysis of the observation and interview data. We conclude with a discussion of the anticipated contribution of this study: insights for capstone design instructors as they support engineering students' integration of mathematics and mathematical thinking into their design practices.*

Index Terms — *engineering design, capstone, mathematical thinking*

INTRODUCTION AND RELEVANT LITERATURE

Design is essential to all branches of engineering—mechanical engineers may design a car or an airplane, an industrial engineer may design a supply chain, a civil engineer a bridge, a chemical engineer a process for converting resources into energy. Each of these engineers completed a rigorous mathematical program to earn the engineering degree, and it may be that it is the mathematics that distinguishes these engineers from other designers [1], but these engineers' use of mathematics is far less obvious than their design behavior.

Because design is foundational to engineering, much research has been conducted to try to understand design practices. With this, a number of coding schemes have been developed to describe the different types of activities that designers engage in. This study will rely on a design activities coding scheme developed by Atman and Bursic's content analysis of engineering textbooks [2]. These activities are presented in Table 1.

Design Activity	Description
Problem Definition	Define what the problem really is, identify the constraints, identify criteria, reread problem statement or information sheets, question the problem statement
Gather Information	Search for and collect information
Generate Ideas	Develop possible ideas for a solution, brainstorm, list different alternatives
Modelling	Describe how to build an idea, measurements, dimensions, calculations
Feasibility Analysis	Determine workability, does it meet constraints, criteria, etc.
Evaluation	Compare alternatives, judge options, is one better, cheaper, more accurate
Decision	Select one idea or solution among alternatives
Communication	Communicate the design to others, write down a solution or instructions

TABLE 1
ENGINEERING DESIGN ACTIVITIES

While the last 50 years have seen a multitude of studies on design behavior, little work has been done to relate engineers' design processes to their use of mathematics, and the work that does exist offers inconsistent messages. Pahl and Beitz [3] maintain that the mathematical formulation of the design problem may lead the designer away from creating an appropriate global formulation of the problem, but offer little evidence on why this may be the case. In contrast, Maher and Tang [4] and Cross and Dorst [5] maintain that development of design solutions can lead to refined understandings of the design problem (and that each new formulation of the design problem furthers the design solution). While they do not specify the part that mathematics may play in the co-evolution of problem and solution spaces, we might suspect that attending to mathematical models is one of the ways that designers develop solutions. It would then follow that developing mathematical models of solutions leads to a refined understanding of the design problem—quite the opposite of what Pahl and Beitz have suggested.

Further evidence of the co-evolution of problem and solution spaces exists in the mathematics literature. In their study of conceptual learning in mathematics, Greeno, Sommerfeld and Wiebe [6] found that groups created problem and situation models as they worked on open-ended projects based in real-world contexts. The students' problem and situation models co-evolved until the group finished the project, much in the same way that designers' problem and solution spaces co-evolve. Similarly, Stylianou [7] found that expert mathematicians' visualizations and analyses of problems co-evolved throughout their problem solving processes.

Schoenfeld [8] provides another perspective on the use and learning of mathematics in his discussion of mathematical thinking. He considers thinking mathematically as having a mathematical point of view—valuing the processes of mathematization and abstraction—and having competence with the "tools of the trade." He identifies five elements that constitute a framework for understanding mathematical cognition: the knowledge base (e.g., calculus), problem solving strategies or heuristics, effective use of one's resources (or ability to monitor use of resources), mathematical beliefs and affects, and engagement in mathematical practices. These five aspects of thinking mathematically provide a framework that can be used to understand how engineering students use mathematics—or "think mathematically"—while solving design problems.

However, some members of the engineering design community consider problem solving and design to be quite different. Design is recognized to encompass ill-defined problems, while "problem solving" generally involves solving well-defined problems [9]. One difference here is that well-structured problems can be reduced to an algorithm but ill-structured problems cannot [10]. Additionally, design is considered a creative endeavor while problem solving emphasizes systematic techniques to solve something that needs a solution [9]. Schoenfeld's definition and description of mathematical problem solving, however, seems to resemble the design community's definition of design. Because of the differing definitions (both across and within communities) of "problem solving," it is difficult to automatically apply findings from studies of mathematics in the problem solving setting to mathematics in the design setting.

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In this document we describe a qualitative study that is part of a larger attempt to understand the role of mathematics in engineering design. In this particular study, we investigate through observations and interviews the way that engineering students use mathematics while engaged in their capstone design project. Specifically, we focus on the question: How do engineering students engage in the five aspects of mathematical thinking identified by Schoenfeld while working on their capstone projects?

In the next section we explain our rationale for choosing the capstone course and for choosing to focus on Industrial Engineering students for the observation portion of the study. We then present our data collection strategy and procedures, including a description of the students that participated in the study and a description of the data analysis methods that we used. Next, we present evidence of engineering students' mathematical thinking activity for each of the five aspects, with design activities embedded in this discussion. Finally, we present some suggestions for future work as well as some ideas for supporting engineering students' mathematical thinking.

WHY THE INDUSTRIAL ENGINEERING CAPSTONE COURSE?

Focusing on engineering students affords us information about engineering students' preparedness to enter industry. During the capstone design project, engineering students are given the opportunity to integrate all of the material that they have studied throughout their undergraduate careers by applying it to an authentic problem. Other salient characteristics of the capstone project are the teamwork and the length—a two-quarter sequence allows students to experience the full extent of the conceptual design process, while bounding the study to an analyzable length.

The qualitative study focused on the Industrial Engineering capstone for the observation portion of the study. Industrial Engineering was chosen for two main reasons: practical concerns and partnerships with industry. The researchers' affiliation with the Industrial Engineering program provided access to the research site and knowledge of Industrial Engineering practices that facilitated analysis of the students' activities. The Industrial Engineering capstone course was also chosen for its strong ties to industry—the Industrial Engineering projects involved industry clients.

The qualitative study is composed of four main components: (i) observation of one team of five Industrial Engineering students throughout the course of their capstone project, (ii) interviews with a subset of the team, (iii) analyses of documents collected from the team and (iv) interviews with engineering students from other engineering disciplines, to ensure that the investigation was not limited to an Industrial Engineering perspective.

DATA COLLECTION STRATEGY AND PROCEDURES

As previously mentioned, the primary aim of the study was an observation of a team of students as they worked on the capstone design project. The observations focus on activities related to Atman and Bursic's design step coding scheme [2] and Schoenfeld's description of mathematical thinking [8]. While many studies of engineering design have taken the form of Verbal Protocol Analysis [11], there is precedent for observational studies in engineering design, such as Bucciarelli's study of engineering design as a process of achieving consensus [12].

To accomplish this, the first author and primary researcher collected observation notes using a Tablet PC, recorded students' conversations using a digital audio recorder and photocopied students' design documents as artifacts of mathematical activity for a subset of the team's meetings—22 in all. Two of these meetings involved the company client, and one was a consultation session with an instructor from the department.

To complement the observations, the researcher interviewed students to elicit further insight into the students' design behavior, similar to Cross and Clayburn Cross' use of interviews to elicit design expertise [13]. Four of the Industrial Engineering students were interviewed for approximately half an hour, at different points during the project to accomplish coverage of the activities while minimizing students' level of effort. An additional four students from other engineering departments were also interviewed. Like the observations, the interviews were recorded using a digital audio recorder and notes were collected using a Tablet PC. The interviews inform interpretation of student behavior as well as decisions on which activities to observe.

Research Participants

This study focuses on one team of five Industrial Engineering students working on an optimization and scheduling problem. Other College of Engineering students were also invited to participate in an interview-only phase of the study. Two Aeornauticals & Astronautical Engineering students, one Materials Science & Engineering and one Chemical Engineering student agreed to be interviewed.

The Industrial Engineering team consisted of four male students (B, D, F, G) and one female student (H). In addition to replacing the students' names with single letters, details of the team's project have been minimized in order to protect the students' confidentiality.

The Industrial Engineering Team

The group of five Industrial Engineering students who participated in the study was representative of the department demographically and academically. The particular team of five students was selected based on students' willingness to participate in the study. Two of the students were involved in the student chapter of the professional society associated with Industrial Engineering and two were members of the Industrial Engineering honor society. Two of the students worked part-time jobs on campus; the first group meeting that the researcher observed was held at this workplace while the student was working. Another worked for the company client—consequently several meetings were held on-site at the company. This afforded the group increased access to information. Most of the meetings that the researcher observed, however, were conducted in the IE undergraduate computer lab and the IE student lounge.

The Industrial Engineering team worked on an optimization and scheduling problem, and competed against four other teams of their classmates for the capstone design competition prize: $1000. The other four teams in the course worked on optimization, factory layout, human factors, and time study problems.

Participants from Other Engineering Departments

In addition to the five industrial engineering students, a Chemical Engineering student (ChemE1), a Materials Science and Engineering study (MSE1) and two Aeronautical & Astronautical Engineering students (AA1 and AA2) participated in the study. These students were interviewed only. Unlike the Industrial Engineering teams that each worked on a different project with a different client, the Chemical Engineering students all worked on the same problem, the Materials Science and Engineering students all worked on the same problem and the Aeronautical & Astronautical Engineering students all worked on the same problem. Each team of Chemical Engineering students designed a lab chip for eventual sales to an education market. This chip would allow the user (e.g. high school students) to place drops of solutions on the chip with a goal of determining what an unknown solution is. Each of the teams of Materials Science and Engineering students worked on an airplane fuel plug project, where the main concern was choosing a material for the fuel plug, though each group made unique decisions about what material to use. Finally, all of the students in the Aeronautical and Astronautical Engineering capstone course worked on a reusable launch vehicle project, with each student working on a different aspect of the project.

Data Analysis

The data resulting from this study includes observation and interview observation notes, audio recordings of the observations and interviews and copies of students' work (interim design documents). The data was analyzed based on Atman and Bursic's design activities [2] and Schoenfeld's description of mathematical thinking [8]. While Atman and Bursic provide a formalized and tested coding scheme, with consistent results [14,15], Schoenfeld identifies five fundamental aspects of thinking mathematically (core knowledge, problem solving strategies, effective use of one's resources, having a mathematical and engagement in mathematical practices) but does not present a clear, tested coding scheme corresponding to these five aspects. Therefore, his descriptions of mathematical thinking and aspects of mathematical thinking were used as guidelines in conjunction with a categorization approach similar to the methods described by Strauss and Corbin [16]. Functionally, excerpts from observation notes and interview notes from a subset of the data (data from the first quarter) were grouped in order to form mathematical thinking activity categories and the groups that had emerged were then mapped to Schoenfeld's descriptions. One of the emergent themes was then explored using second quarter data. While most of the coding was done by a single coder, one interview transcript was also coded by two other coders to check inter-rater agreement.

The results of the data analysis are presented in the next section of the paper. Aspects of mathematical thinking are explicitly addressed in this section, while the design activities are woven into these discussions in a capitalized form—for example, the term Gather Information refers to the term "gather information" defined in Atman and Bursic's coding scheme and presented in Table 1.

EVIDENCE OF ENGINEERING STUDENTS' MATHEMATICAL THINKING

In this section we present the themes that emerged from grouping of a subset of the data (data from the first quarter only) according to the five aspects of mathematical thinking described by Schoenfeld [8]. These themes are listed in Table 2. We further explored one of the themes—"dealing with uncertainty"—with the remainder of the dataset. This theme was chosen because it was so prevalent in both the observation and interview data, and because there are a number of opportunities to support engineering students in dealing with uncertainty.

Aspect	Theme
Knowledge Base	Content knowledge
Problem Solving Strategies	Guess & Verify, Separating into Smaller Problems
Effective Use of Resources	Resources: tools and experts
	Monitoring of use of resources: planning and choosing the right approach
Beliefs and Affects	Students beliefs: mathematics is only about the content knowledge, mathematical is a tool, mathematics is a form of thinking
	Cultural beliefs: mathematical ability is innate
Mathematical Practices	Having a mathematical perspective and a mathematical vocabulary, dealing with uncertainty, and estimating

TABLE 2
EMERGENT THEMES GROUPED BY THE FIVE ASPECTS OF MATHEMATICAL THINKING

Knowledge Base

Schoenfeld's first aspect of mathematical thinking is core knowledge. Generally, when students were interviewed about their math courses, they talked about the content knowledge that they learned. Occasionally they also talked about the strategies or practices they learned in their math courses, but their tendency was to think about math in terms of core knowledge rather than thinking process. As we further explore engineering students' knowledge bases, relevant questions are: what content knowledge do the students have? What content knowledge are the students able to access and apply to the project they are working on?

During the first quarter, the team incorporated basic mathematical content knowledge into their capstone design process, though they did not yet incorporate more advanced topics such as calculus or differential equations. The team relied on fundamental mathematical concepts, such as taking averages and changing bases (observation notes, meeting 2, February 20, 2004) as well as regression and statistical analyses (team document: Final Proposal 2).

Problem Solving Strategies

Schoenfeld grounds his discussion of problem solving strategies, or heuristics, in Pólya's book, How to Solve It. He notes that the "strategies" that are taught in schools via a series of rote exercises are more algorithms than the heuristics described by Pólya. In this section, we explore the two main strategies that the emerged from the clustering of observation and interview data: "guess and verify" and "separating a large problem into smaller problems".

Guess & Verify

As the team looked at the data that their client had given them, there were some inputs that needed additional explanation. For this, they used a "guess and verify" strategy that Schoenfeld's describes as part of mathematical problem solving [8]. Generally the team guessed at the meaning of the inputted values and then planned to verify their meaning by asking either their client contact or one of the other employees from their partner company. "Not a single [entry] touches 60 [the] actual time – another thing to ask the [employees] about" (observation notes, meeting 2, February 20, 2004).

Separating into smaller problems

Another strategy that the team used was to decompose the problem into smaller, more manageable problems. D, F and H all mentioned that they had learned this strategy from their math classes and were applying it to their capstone design project. D talked about the strategy in terms of "making difficult concepts simpler" and "breaking down the problem" (interview notes, D1). F used the term "decomposing" and described the process as "taking one step at a time" (interview notes, F1). Finally, H talked about how in a math class she had learned a strategy of taking things "step by step" starting with identifying "what you need to find and how to find it" she continued to explain that the team had also done this in their senior design project "figure out what you need; set [the problem] up; [if you] don't know how to solve for ___, take [it] step by step" (interview notes, H1).

Effective Use of Resources

While solving mathematical and engineering design problems, people are limited in their mental processing by working memory load limits. The problem solver must be aware of these limitations and be cognizant of the how they use resources, given these limitations. This monitoring and control of resource use is one aspect of metacognition. While expert problem solvers and designers exhibit monitoring and metacognition [17], it is less common for students to do so.

In this section, we first identify two resources available to the students and then discuss both their use of these resources and two ways that the students monitored their use of resources.

Resource 1: Tools

One of the ways that engineering students conceptualize mathematics is as a tool for solving problems [18]. In some instances, engineering students use mathematical tools such as Excel to solve problems and in other cases engineering students use mathematics itself as a tool to perform Feasibility Analysis, Evaluate a solution, make a Decision or Communicate a design as evidenced in the following episodes.

The team used Excel and MapPoint as mathematical tools in their analysis process. During an interview, F expressed this conceptualization in terms of using the following mathematical tools during the team's project: "multiple metrics, statistics, computer tools- random distributions, Excel- time simulation- [to find out] what's fastest? [using] random partition- partition then sort" (interview notes, F). Here the time simulation that F commented on was a simulation that D ran using Excel to model three alternative solution and then determine which of the three was fastest. In this case, F saw D using Excel as a mathematical tool to make a Decision about which of three options was optimal. D explained the simulation as a "general model- each process one loop… different alternatives change queue time" (field notes, meeting 4, March 10, 2004). Another mathematical program that the team used as a tool was MapPoint- a mathematical tool that allowed them to determine travel times and commute times. B: "MapPoint is the commercial version" H: "traffic and stuff" B: "made for industrial engineers- travel times, commute times" (audiorecording, meeting 1, February 17, 2004).

Beyond Excel and MapPoint, the engineering students used mathematics itself as a tool. Part of mathematical thinking is being able to tell if the value that your program gives you is reasonable. During meeting 4, F tested the validity of the database by comparing some of the outputs of a query to the values expected (Observation notes, meeting 4, March 10, 2004) In this sense, mathematical thinking was an Evaluation tool for the team. H suggested that math can help one "figure out further what you can and cannot do and what a feasible solution is" (interview notes, H1) and the team often referred to criteria requirements to Evaluate their solution. For example, during meeting two F asked the team "did we decide about criteria requirements?" the team talked about criteria for a while, then D commented "[we] have to have criteria to know how to meet goals" (observation notes, meeting 2, February 20, 2004). Estimation, discussed in a separate section later in the paper, is another mathematical tool that the team used for Evaluation and Feasibility Analysis.

The team also used mathematics as a Communication tool to persuade their audience and justify their assumptions and suggestions. During an interview, D suggested that if an employer is concerned about a recommendation, he could back up his ideas with numbers (interview notes, D1). In his interview, F thought that the team now needed to "evaluate from a business perspective," which mainly meant they needed a cost justification (interview notes, F1). The team also discussed justifying their solution from a numerical perspective—"volume can justify placement of [the building]" (observation notes, meeting 2, February 20, 2004).

Resource 2: Experts

Beyond mathematical tools such as Excel, Arena and MapPoint, the team drew on expert opinion as a resource. For example, the team consulted with the simulation instructor for extra guidance, after realizing that another team had gotten feedback on their simulation from this instructor (observation notes, meeting 4, March 10, 2004). The team also met with their course instructor and their company client throughout the project, and received guidance and feedback from both of these sources. Additionally, the team recognized that the researcher observing them could be a potential resource for them. However, during the recruitment and consent process the researcher had explicitly informed the team that the researcher would "not be able to give any type of extra assistance on the projects" (excerpt from the recruitment email sent to the Industrial Engineering students enrolled in the capstone project course).

Evidence of Monitoring: Planning

In his coding scheme for mathematical problem solving, Schoenfeld identifies planning as one of the activities involved in monitoring [8]. The students also identified planning as a key part of their design process. The team applied an analytical thinking to the process of writing out the problem, identifying the data that they needed and gathering the data as part of their "planning" process (interview notes, H1). This analytical way of thinking is something that H identified as having learned in her math courses.

Additionally, the team had a clear sense of planning in terms of knowing when they wanted to apply mathematical techniques. They had a "goal for [the] end of [this] quarter: gather info" (observation notes, meeting 3, March 1, 2004) and they wanted "to dive in to number crunching when spring starts" (observation notes, meeting 4, March 10, 2004).

Their planning was motivated by a course requirement: a PowerPoint presentation that they needed to present at the end of the first quarter, as part of their project proposal. In preparing the presentation, they considered the mathematical content knowledge and the mathematical tools they would be using for the project. They identified Linear Programming, Queueing Theory and Operations Research as the content knowledge they would need during the second half for the project, but they did not "know what tools [they] need"—aside from simulation (observation notes, meeting 2, February 20, 2004).

Evidence of Monitoring: Choosing the right approach

Many of the engineering students recognized that when solving both math and engineering problems, there are often multiple approaches that one can take. They identified "choosing the right approach" as a strategy, so we could group "choosing the right approach" in the Problem Solving Strategies category. However, we consider this strategy to be evidence of monitoring because the problem solver consciously chooses an approach rather than following the first one considered.

D first identified "choosing the right approach" as a mathematical practice that he had recognized in his math instructors—in particular, the instructors used a variety of approaches to solve proofs (interview notes, D1). F, however, recognized that in his math classes, he had learned to look for the right "trick" to solve problems, but in the real world, there are "multiple ways to try to prove something. Some directions you go won't get you anywhere," (interview notes, F1). In the context of the capstone project, the team found that their client had given them access to a great deal of data, and needed to figure out how to best use it, "F: want to start data analysis; H: what are we going to do with all of this?" (observation notes, meeting 2, February 20, 2004). The team had many options of how to analyze their data, and needed to successfully identify an approach that would afford them useful information.

At times, F played the role of making sure that the team was choosing the right approach, often by asking questions. During one meeting he prompted the team to justify the approach they were using: "how are you going to…" (audio recording, meeting 1, February 17, 2004). After explaining their approach, his teammates identified a possible problem: most of the data they were analyzing was from 2002, but data for one of the employees was from 2003. In terms of design activity, F asked the team to Evaluate their work, which led them to Gather Information from the database, which led them to identify a problem.

Beliefs and Affects

Another aspect of mathematical thinking is beliefs and affects—students' beliefs and attitudes about mathematics influence how and when students utilize mathematical knowledge. Additionally, students' are influenced by beliefs held within their culture. The literature provides some insight into the cultural beliefs about mathematics. For example, Americans tend to believe that mathematical ability is innate; you either have it or you don't [19].

One of the beliefs that many of the engineering students expressed was that mathematics is synonymous with mathematical content knowledge. As mentioned in the Knowledge Base section, when asked what they had learned in their mathematics courses, many students simply recited different algorithms or rules they had learned rather than problem solving strategies or mathematical practices. Also, as mentioned in the Tools section, many engineering students express a belief that mathematics is equivalent to a set of tools.

Some of the engineering students did, however, have a broader sense of mathematics. ChemE1 described mathematical thinking as a form of problem solving and a way to approach physical problems and then discussed mathematics as content knowledge (interview notes, ChemE1). H also said that she learned to solve problems analytically and think about numbers (counting, mental math) from her math classes, but that she learned more in high school than in college (interview notes, H1). Finally, MSE1 said that math is problem solving and that math involves rigorous thinking for proofs (interview notes, MSE1).

Mathematical Practices

Expert mathematicians engage in mathematical practices established within their community and have competence with the "tools of the trade." What mathematical practices have students learned through their experience in their courses? Did the students learn "school mathematics" in their courses, or were they enculturated in a learning environment where they were able to engage in "real" mathematical practices? Schoenfeld identifies a number of authentic mathematical practices that engineering students may have been exposed to (e.g. interpretation, sense-making, looking for patterns, making conjectures, defending claims mathematically) [8].

Mathematical Perspective & Vocabulary

One aspect of engaging in mathematical practices is having a mathematical perspective. As the previous sections suggest, the engineering students were able to recognize opportunities to use mathematics to further their design project. The students were able to recognize how to apply mathematics because they looked at the problem with a mathematical perspective. Additionally, they displayed a mathematical perspective—a tendency to mathematize—through their mathematical vocabulary.

In formulating the problem, at one point G translated the problem into mathematical language. "L: want to know traffic; G: know Xs and Ys; D: average commute…" (observation notes, meeting 2, February 20, 2004). The team did not continue to formulate, the problem as a series of equations, but did recognize the problem with a mathematical perspective as G applied the appropriate mathematical terminology. Because of the students' similar math backgrounds, they were able to use this terminology in their Communication. During an interview, F expressed frustration in "talking to non-math person…can't talk to some people [because they are] on a different level" (interview notes, F1).

Uncertainty & Estimation

Another mathematical practice identified by Schoenfeld is "coming to grips with uncertainty." Both the Industrial Engineering students and the other engineering students interviewed met with uncertainty in the projects that they were working on. One of the problems the Industrial Engineering students faced was that they did not know details about the partner company's current operations. "D: how do they do this now? This is frustrating! These times make a huge difference!" (observation notes, meeting 11, May 10, 2004). One of the reasons that they wanted to know more about the company's current operating mode was because of the tight time schedule that constrained their solution. "D: main problem: can guys finish in time? ... B: just need to know current solution!" (observation notes, meeting 11, May 10, 2004).

Often they were not able to simply "come to grips with" uncertainty, in the sense of acknowledging that uncertainty would remain in the project. Rather than model the uncertainty mathematically, the students generally tried to gather more information to eliminate the uncertainty—either from their clients, instructors, textbooks or even off the internet. "D (looking at GMC website) width of each parking spot should be 7 feet" (observation notes, meeting 16, May 26, 2004).

The other main method that the engineering students used for dealing with uncertainty was estimating. During the project, the team frequently used estimation, commented on their use of estimation, and experienced tension between using estimates versus using precise numbers.

In an interview, F identified estimation as one of the ways that the team used mathematics for their project (interview notes, F1). One of the ways that the team used estimation was to estimate the amount of time it took for employees to perform certain tasks. During a meeting D emphasized the team's need to complete a thorough cost analysis, and to complete this, they need to know the costs associated with a particular task. To calculate the costs, the team needed to know how much time the employees spent on the task so that they could then multiply the amount of time by the employees' hourly wage. L suggested estimating the time to complete the task—estimation gives us something to follow" (observation notes, meeting 3, March 1, 2004). The team also used estimation during the second quarter to get a sense of the cost of real estate. During meeting 16, B and F were trying to determine the minimum amount of space they would need for the facility they were designing.

Part of the need for estimating arose from a need to Gather Information—actual numbers—from the client. "B: get from K specifications of build size- need to know volume first- can guesstimate" (observation notes, meeting 2, February 20, 2004). Estimation also arose in the midst of the team's Communication about the different pieces of the project each team member worked on. H explained that "everyone knew about different parts, so estimate to get big picture." (interview notes, H). It is important to note that the team did distinguish their estimates as being estimates: "get more accurate info for report- specify that we're using estimates now" (observation notes, meeting 2, February 20, 2004).

However, the fact that they decided to "specify that we're using estimates now" illuminates the tension between estimation and precision that the team encountered. During the first observed meeting, B and D suggested that a problem with the team's formulation was the lack of precise distances and times, "B: that's the problem … D: distance… is a rough estimate" (observation notes, meeting 1, February 17, 2004). Later, they reiterated the team's desire for precision, D: "pretty much have all the components, just need" B: "data" (audio recording, meeting 1, February 17, 2004). By the end of the meeting, the team had plans for how to be more precise: talking to the client and observing the employees (observation notes, meeting 1, February 17, 2004). The tension between estimating and being precise continued into the second quarter. During the second quarter, the team felt a greater need for precision as they needed to come up with final recommendations. "D: no metric to say how long 50 more units will take B: Just say an hour D: That was the whole point L: can do a rough estimate D: do you know much estimation costs? Time is so important – has to work perfectly!" (observation notes, meeting 16, May 26, 2004). In the end, the feasibility of the team's solution rested on the cost of real estate— the estimate that they arrived at in meeting 16—rather than a precise number.

Despite the general desire for more precision, the team did recognize some instances where an approximation was more useful than figures that were too precise. For example, in looking at data on amounts of time it took employees to perform tasks, D commented that the output they had was "too detailed" in the form of daily summaries and that weekly summaries would be more useful (observation notes, meeting 4, March 10, 2004).

The other engineering students interviewed about their capstone projects also used estimation or "approximation" methods to deal with the uncertainty in their projects. ChemE1 talked about using "approximation" methods to gauge the financial market for the chip his group was designing. AA2 also talked about how for his project, he needed to make approximations based on earth's shape while designing the altitude control system and navigation control system lead.

FUTURE RESEARCH: HOW ARE ENGINEERING STUDENTS' MATHEMATICAL THINKING ACTIVITIES INTEGRATED INTO THEIR DESIGN PROCESSES?

As previously mentioned, this qualitative study is part of a larger attempt to understand the role of mathematics in engineering design. For her dissertation, the first author will be investigating the role of mathematics in engineering from a number of angles, including both qualitative and quantitative studies and lab-based and natural-environment studies. One of the opportunities for further exploration is investigation of the relationships between mathematical thinking and design activities.

The engineering students that were interviewed and observed for this study offer some initial evidence of how engineering students use mathematical thinking while engaged in the design activities listed in Table 1. The students in this study used mathematical thinking to define the problem that they were working on when they identified the problem as a math problem and talked about the problem with a mathematical vocabulary. They also "mathematized" the current state of the system they were trying to improve. They analyzed data to Gather Information and to determine if the current solution would work (Feasibility Analysis) and if it was a good solution (Evaluation). They also used results of their analysis to justify their approaches and solutions to themselves (Evaluate) and to others (Communicate). Additionally, the students used their estimation technique to perform a quick Feasibility Analysis and Communicate design decisions. Finally, they used mathematics as a tool while make Decisions.

However, there are still opportunities to further investigate the relationship between mathematical thinking activities and design activities. For example, we might further investigate Pahl and Beitz's suggestion that the mathematical formulation of the design problem may lead the designer away from creating an appropriate global formulation of the problem [3]. We might also investigate the relationship between engineering students' ability to monitor their mathematical activities and their ability to monitor their design activities.

ANTICIPATED CONTRIBUTIONS

Understanding how engineering students use mathematics on the capstone project affords us information to improve both engineering and mathematics education. First, engineering education as a discipline benefits from insight into how engineering students are able to integrate mathematics into engineering design. Additionally, engineering capstone courses can directly benefit from this characterization of students' mathematical and design behaviors. The results of this study can also inform design education as a subset of engineering education as we gain new understanding related to design activities, how engineering students practice particular design activities, and how mathematics supports design in specific ways. The results of this study also benefit mathematics education as we gain further understanding of how mathematics is actually used and how students conceptualize mathematics.

Comment [MEC1]: Be more specific---what do the results that were presented here speficifcally suggest for engineering education?

The results of this study will likely provide further opportunities for both exploring and supporting the integration of mathematics and engineering design. These opportunities include, but are not limited to, the development of tools and curriculum to support engineering students' integration of mathematics and engineering design, and further research on the role of mathematics in design. This research can be extended to practicing engineers as well as pre-college mathematics students. The study can also be extended to mathematics students, who are not engineering majors, and the results of this study can inform the development of laboratory based and quantitative studies of mathematical thinking in engineering design.

In particular, we noticed that engineering students are not fully aware of the ways that they use mathematics, and at times are unable to apply the mathematical skills they have learned. In general, they had an incomplete understanding of mathematical thinking, as evidenced by the mathematical practices they were missing. For example, they were unable to come to grips with uncertainty. They continually grappled with tension between estimation and precision. This study suggests possibilities for future research and opportunities to develop supports (e.g. curriculum, tools).

As a future research possibility we can explore the tension between estimation and precision in other design contexts. As an alternative approach, one question arising from the observed tension between estimation and precision is: are there differences between mathematical practices and engineering practices, such that one discipline accepts estimates more readily while the other requires precision?

To support students as they grapple with uncertainty, we might modify the curriculum to include more stochastic processes and methods, and more emphasis on dealing with uncertainty through mathematical models. A number of possibilities for tools to support use of estimation or more general integration of math exist; in this document we limit our discussion to one possibility: a simple rubric. Observations of the Industrial Engineering students showed that a simple one page rubric from their capstone instructor supported their integration of socio-environmental considerations into their design—periodically, the students would turn to the rubric to monitor their progress on the project. While looking at the rubric they were reminded to consider how their solution would affect people and society and how their solution would impact the environment. Rarely did they make these types of considerations without the rubric in front of them. Following this, we could add to the rubric or create another paper tool that prompts students to consider when to estimate and when to be precise, and other ways to integrate mathematics into their design.

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