MISLEADING SHORTCUTS IN COMPOUND 3D VECTOR ROTATIONS

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Abstract

The idea of this research was inspired by the problems in vector rotation that are faced by most of the undergraduate students who study computer graphics courses after they have studied a course in linear algebra. In one particular computer graphics mid-term exam, all of the fifteen candidates failed to conceive the angles of a compound vector rotation that could make the X-axis parallel to the diagonal of a cube of unit side length. Interestingly, all of the candidates took the exact same wrong path and got to the same wrong solution and understood their mistake when the correct solution was shown to them. This verified that it was not an error in calculation but some weakness in conceptual approach. Moreover, there are various different analytical problems from the same subject in which the candidates are required to overcome their misleading visualization and use the appropriate scripts and concepts for solving them correctly. The visual misconceptions get triggered by tempting shortcut approaches which seem the correct choice usually because of the simplicity of the problem. When using the shortcut approaches, candidates usually overlook some concepts or their attributes that are involved in the problem and so they apply the inappropriate procedures without noticing that the script they are using does not qualify to be used to that particular problem. This research found the cause of this commonly occurring problem and also provided an alternate approach of teaching 3D compound vector rotation to help students avoid misconception when solving such problems.

Introduction

Researchers have presented various different views regarding the process that takes place inside a human brain when learning is taking place [1,6,7,8]. Learning new concepts is a complex but useful process. According to Ausubel, It may either be achieved by meaningful learning or rote learning. Meaningful learning builds strong concept maps, in which concepts are properly linked with each other. Rote learning fails during retrieval of knowledge from the existing concepts. If these concepts are not linked with each other properly then a lot of problems of reasoning, inferencing and problem solving arise [1]. In terms of data structures in computer science, meaningful learning is just like a hash map where as rote learning can be said as a linked list.

Over the last few decades owing to high paced advancements in almost every field of life an enormous amount of knowledge is being frequently poured into the existing body of knowledge in this world. In order to help individuals to deal with such huge amount of knowledge we need to develop the intellectual tools and learning strategies so as to empower the learners to become self-sustaining and lifelong learners [4, 5].

When we come to the field of computer vision and computer graphics, both the fields are heavily dependent on mathematics and more specifically on the concept that are usually taught to undergraduates in courses of Linear Algebra. In most of the universities, either the course of computer graphics includes the required mathematical part in itself or has linear algebra courses listed as a prerequisite of it. It has been noticed that in educational institutes where such requirements are not hardly applied, students who have studied relevant mathematical courses tend to perform much better as compared to the students who have not studied such courses previously. This difference is much more prominent in the start of the course which highlights the head start that the first category of students get.

In the elementary Linear Algebra course, students learn a lot of very useful concepts. Starting from the basics, they learn about dimensions in general, vectors and matrices, different types of vectors and matrices, various kinds of operations that could be performed on matrices and vectors and the way of handling different types of vectors and matrices. Students are also taught how the matrices and vectors are related to each other, different ways of representing the data at hand, rules, regulations and shortcut approaches in the subject and certain standard procedures as well that come handy in processing the data. A student that has the background knowledge of this subject definitely has an advantage over the student who does not has any concepts of this subject.

On the contrary, there are certain disadvantages for those who study the Linear Algebra courses that do not have an applied perspective to them or do not lead to a higher level applied algebra course. When all those concepts are bombarded at the students during the theoretical course, the students remain unable to link the concept appropriately and make a concept map that is very similar to the master concept map or the concept map of the instructor. Teaching the course gets easier and easier as the course gets more applied and closer to real life because it becomes easier to give examples that are closer to real life and hence can be visualized easily.

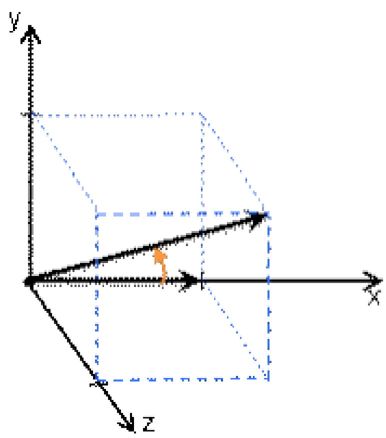
Real Problem and its solution

It was during a midterm exam session of the course "Computer Graphics" that the students were supposed to solve the problem that inspired the basic idea of this research. The question paper was roughly in the order of increasing difficulty and this specific question was posed in the very beginning of the question paper. Although the course was being taught by two instructors in the start and the exam was not equally made by both the instructors but the question does not deal with much of the content of the course.

The problem essentially required the students to calculate the rotations that are to be applied on the x axis to make it parallel to the diagonal of a cube of unit dimensions in 3D(fig). Astonishingly, such a trivial problem was unsolvable for all the fifteen students that were taking the mid term exam. This result raised a few eyebrows and attracted focus. We had to determine where the root cause of the issue lied so that it could be rectified. Since the problem was so trivial with respect to the rest of the course content, the cause of it had to be somewhere in the basics or prerequisite knowledge of the course; Elementary Linear Algebra. This led us to probe the students for deficiencies in their Linear Algebra skills in order to find the exact cause in the lights of the cognitive theory of meaningful learning given by Ausubel and Novak[1].

Meaningful Learning

Learning is a continuous process which adds new information to the existing information repository. If the learner is interested to relate new information with the existing one instead of just adding it, then he is attempting meaningful learning. It means that when knowledge structures are properly organized then it enhances meaningful learning [1]. If one concept is introduced and learner feels difficulty to relate it with the existing knowledge, then perhaps the learner is being pushed towards rote learning. When perceived regularity of an event or object is quite clear in the mind of the learner, then he may feel comfortable to integrate it within the existing knowledge structure.



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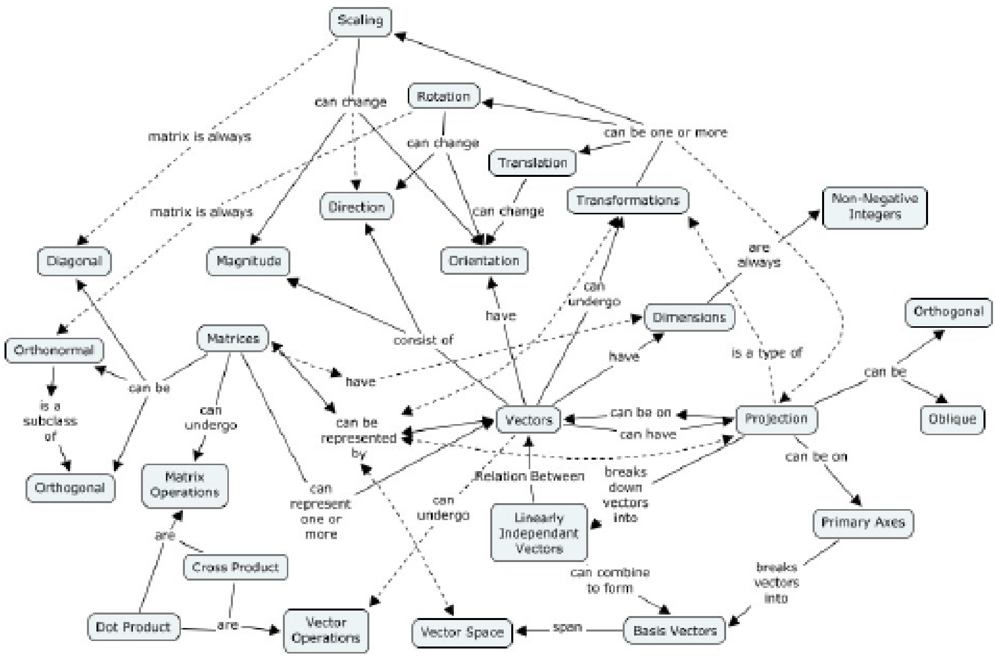
Learning is a continuous process and as this process proceeds, cognitive structure becomes more and more precise and strong [1]. At the early stages of learning, some concepts may be poorly or vaguely understood. So, their perceived meanings are also vague. With the acquisition of new knowledge, those vague meanings start becoming more precise and specific. So, in this continuous learning process, concepts progressively differentiate, and after certain period of time, they achieve a desirable level of specificity [1]. The process of organization of concepts and their linkages carries on all the time during the learning and usage of concepts. Progressive differentiation is the process which refines the concept meanings and associated regularities in cognitive structure with the addition of new knowledge. Sometimes, existing concepts don't need to be refined contextually, but they are reconciled in an integrative fashion with the new concepts to enrich the cognitive structure or concept map[3]. In the process of meaningful learning, direct integration of the new concepts with the existing ones is a very complex procedure. There are some perceptual barriers which resist this integration. These barriers can be easily and effectively crossed, if there are some facilitating concepts called subsuming concepts, in Ausubel's assimilation learning theory [1].
There are two broader categories of the concepts [1]: primary concepts and secondary concepts. Primary concepts are the concepts involving core knowledge. Secondary concepts are supporting concepts which help to acquire new concepts. Ausubel's assimilation learning theory [1] names these secondary concepts as "subsuming" concepts. Subsuming concepts facilitate flow of knowledge though the perceptual barriers and provide a platform so that new concepts may be linked with previous concepts [1,2]. While learning new concepts, not all the concepts are built from scratch but due to the process of subsumption, concepts higher in hierarchy are modified into more specific concepts. In that case, only the new information is updated and all the previous properties of the concepts are maintained as they were. In such a case, to observe meaningful learning that is accurate as well, the concepts have to be modified by adding new information as well as removing irrelevant and inapplicable information. Sometimes, there can be more than one concept maps in the learner's mind in which case super ordinate learning takes place. Super Ordinate learning occurs when new concepts are constructed that pull together and integrate large domains of knowledge that were not previously recognized as intimately related.
It has been observed that meaningful learning gets even more complex and difficult when a learner is simultaneously dealing with problems from multiple domains. This might just be due to the short term memory overflow but no clear evidence exists which explains the prospective issues that might hinder meaningful learning under a situation of dealing with problems that require simultaneous usage of knowledge from multiple domains. In the relevant field, students need the appropriate concepts of visualization of dimensions and their bridging has to be strong with the theoretical operations and concepts that form the solution to the exam problem.

Concept Maps

Concept mapping is a technique for visualizing the relationships among different concepts. A concept map is a diagram showing the relationships among concepts. Concepts are connected with labeled arrows, in a downward-branching hierarchical structure. The relationship between concepts is articulated in linking phrases. Concept maps have their origin in the learning movement called constructivism. In particular, constructivists hold that prior knowledge is used as a framework for understanding and learning new knowledge and hence, a process of meaningful learning takes place. Decades of empirical and qualitative research has verified the efficacy of concept maps for the tasks[3].
We had to develop an expert level concept map to have a reference point when for surveys, interviews, data collection and its evaluation. Fig shows the expert level concept map that students are supposed to have when they study a course of Elementary Linear Algebra.

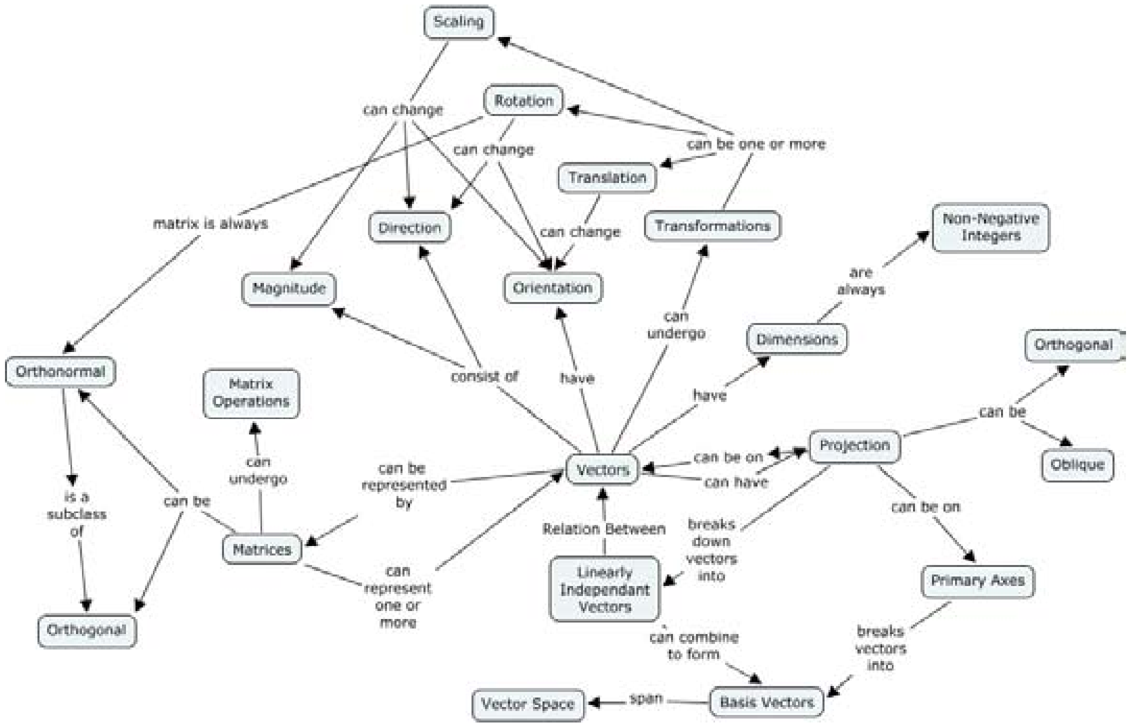
In case of the students, the concept map in fig is not developed at once as it is. In fact, it is developed in parts and merged together only when it has to be used or even then they are used as standalone concept maps. The two different parts of the concept map can be seen easily if all the dotted links are removed and so it is critically essential that the dotted links are formed after the two concept maps have been constructed so that they are patched as one big and more accurate concept map and it is ensured that thorough meaningful learning has taken place and the learner can use the concept map to its fullest with most accurate knowledge structure.



Surveys, Observations, Analysis

A very detailed structured questionnaire was prepared to test the concepts of the learners comprehensively. The questionnaire contained thirty questions regarding the subject of elementary linear algebra. The questionnaire was spread on the intranet and was supposed to be filled on a webpage. A total of forty-one students gave serious responses; students who had studied a course in linear algebra in the past six months. The questionnaire built up to the same exam problem towards the end while testing all the concepts that were involved in the correct approach and the correct shortcut approach. After probing the students for possible holes in their concept maps, a concept map similar to the students' concept map was developed which is shown in fig.
Other than that, think aloud protocols were conducted with five different people to highlight the cause of such inadequate concept maps. Think aloud protocols were conducted for two senior undergraduate students one masters student, one research assistant and a PhD candidate out of which, the PhD candidate and the research assistant were related to the field of computer vision and the rest were related to the field of computer graphics. Both the fields of computer graphics and computer vision are heavily packed with linear algebraic concepts and the interviewed people were considered to be fully relevant to the field of study.

None of the students had studied the Rodriguez formula of rotation around any arbitrary axis. They all tried to use the concepts of projection instead of only rotation as a shortcut to the correct approach. Although the shortcut approach works perfectly fine as well but there were certain misconceptions regarding the missing links in the merged concept map(fig).



The shortcut approach is a creative and uncanny solution to the problem that saves time as well which is of extreme importance from the exams point of view. But because there were holes in the concept map, the creativity led to a disaster. The shortcut approach could only work if the student knows, in full depth, the behavior of projections on vectors as compared to the behavior of other 3D transformations such as rotation. These missing links made the students jump through the procedures that could have made both the approaches equivalent. It is noticeable that the two concept maps are only linked at one point in the center and although it might be called one merged concept map, it actually has two parts with not enough inter linkages. When a student with such concept map tried to take the shortcut approach to use projections to calculate effective rotations, the student lacks essential knowledge that the two procedures are not equivalent and the student also does not have the understanding of the additional procedure that can added on to make both the procedures equivalent and hence the outcome is faulty.

It can be the issue that when learning new concepts of transformations, for every new concept, the students copy the higher level concept and modify its properties to form a new concept. Students seem to be unable to create a few properties and more importantly, rub away the properties of the parent concept that are not necessarily applicable in the case of a lower level concept. For instance, the property of rotation is that it cannot vary the length of the vector that it is applied on, is not clear of most of the students. Students think that like a common property of all transformations, rotation is supposed to change the direction and that is all what their concept map provides them with.

The order of matrix multiplication is critical. It turned out that a major portion of the surveyed students do not have the concept that the order of applying the transformations causes errors in the end result of a matrix multiplication. In reality, the order of applying the transformations change the result of the transformation horrendously and most of the times, this error is undetectable. This helps to explain the error made by students in the sample problem as they think that the order of transformations does not affect the transformations.

Conclusion and Suggestions

As stated earlier, learners tend to make two different concept maps. The two concept maps are constructed independently and are attached together in such a way that some links are missing between the two concept maps. With the missing links, it becomes virtually impossible for a student to solve the problem with the shortcut approach. Special emphasis has to be laid upon developing a better concept map. It can be a whole different way of developing the concept map as a single concept map altogether. Alternately, extra work can be done in the linking of the two concept

maps and the development of the dotted links that are missing in the students' concept maps.
Students think of all the transformations as similar in the first place and they assume some links true for all transformations (such as size and direction conservation). They do not remove the unnecessary links because they are never stressed upon in detail. Either those links can be removed by the instructor or negative linkages can be developed so that when the concept of rotation is accessed by the student, it comes clear that it cannot modify the magnitude of the vector as a rule.
Runtime answer checking is going on during the solving of exam problems. There is a perception in students that simple round figures in the answers are more probable to be a correct answer. This leads to a short circuiting in the process of rationalizing the process in a student's mind and they make errors. During exams, when students are dealing with round figures in mathematical procedures, their confidence in the solution gets reinforced and biased. Students should be given exposure to more actual figures so that this false expectation is eradicated and the rechecking procedure takes place no matter what kind of figures show up in the solution.
Moreover, it is suggested that when dealing with 3D algebra, students should be provided with some kind of real mechanism to help them visualize the problems. These can be magnetic linkages, blocks or even three wires placed orthogonally and linked together at one point to mimic a three dimensional space. The problem faced by the students during the mid-term exam was in fact triggered by errors in visualizations as well.

References

01. Novak, J., D., "Leaning, Creating and Using Knowledge", Lawrence Erlbaum Associates, Inc., 1998.
02. Polya, G., "How to Solve It: A New Aspect of Mathematical Method", Princeton University Press, 1988.
03. Novak, J., D., "Concept maps and Vee diagrams: two metacognitive tools to facilitate meaningful learning", Springer Netherlands, 2004.
04. Mintzes, J. J., Wandersee, J. H. & Novak, J., D., "Teaching Science for Understanding: A human constructivist view", American Press, 1998.
05. Mintzes, J. J, Wandersee, J. H. & Novak, J., D., "Assessing Science Understating: A Human Constructivist View", American Press, 1998.
06. Ausubel, Novak, J., D., & Hanesian, H., "Educational Psychology: A Cognitive View", Holt, Rinehart & Winston, 1978.
07. Anderson, J., R.,"Cognitive Psychology & Its Implications", W.H. Freeman and Company, 1985.
08. Bransford, J., D., Brown, A., L., & Cocking, R., R., "How people Learn: Brain, mind, experience, and school", National Academy Press, 2000.