Teaching of Mathematics in Higher Education

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Abstract *- Logic and deductive thinking as well as formal and precise language characterize mathematics as a powerful tool in scientific and technological development as it offers conceptual elements that represent reality phenomena. Thus, teaching mathematics in Higher Education aims at developing forms of behavior or cognitive competency related to the formalizing, organization, logic reasoning and elaboration of models to represent the properties of situations by means of mathematical concepts. Knowledge of behavioral analysis, and more specifically the concept of behavior, seen as a complex interrelation with the environment, may help find out what kinds of behavior need to be developed when teaching mathematics in order to prepare engineers to deal with a reality that is strongly affected by advances in computer science and technological development, and to achieve significant results. From information gathered from twenty-two subjects that work in Mechanical Engineering, we have identified and characterized, although still partially, mathematical behaviors that mechanical engineers need to show in order to successfully deal with daily routines in their professional environment. Analysis and interpretation of data also indicate that it is necessary to examine and clarify the relationship between: phenomena, phenomena representation, mathematical phenomena, mathematical language, cognitive competence for dealing with phenomena, cognitive competence for dealing with phenomena representation, cognitive competence for determining teaching conditions that enable the establishment of relationships, cognitive competence for evaluating to what extent one is able to establish relations with his/her professional environment, and mathematical behavior necessary for the proposition of a kind of teaching technology for teaching the establishment of relations with the environment having the "mathematical knowledge" as a starting point, that is, for teaching mathematical behaviors.*

Key words*: Teaching of Mathematics in higher education; Mathematical Knowledge; Methodology for the teaching of Mathematics in Higher Education; Objectives of the teaching of Mathematics; in higher Education; Mathematical Behavior; Cognitive Competency in Mathematics.*

In general lines, higher education is nowadays being developed as a series of activities based on information about operations, definitions, results, techniques, manipulation of software, and other procedures. That is to say, higher education is centered on information and results of a "process of building knowledge." The conception of knowledge behind this view of teaching is that knowledge is mainly a product that needs only to be "transmitted" (without considering the inadequacy of the metaphor) by means of speeches and expositions that don't take into account the process of transformation of that knowledge into forms of behavior important to society. On the other hand, this view reduces the process of teaching to activities and operations performed by teachers and students: teaching lessons, operating equipment, watching films, lectures and seminars, solving problems, manipulating symbols, following rules and instructions, making calculations, writing papers, doing experiments, and what is now so fashionable, operating software.

What is the relationship between such activities and the result that is desired, the students' learning? If teaching is understood as "teaching lessons", "presenting information" and "proposing activities", then the teacher teaches even when students don't learn. If learning means "attending classes", "following instructions" and "performing tasks", students may learn even if the activities performed don't qualify them to work "successfully" with situations of their academic or professional environment. The teaching-learning process seen like this aims more at fulfilling academic rituals than at qualifying individuals to have a kind of performance that produces interesting results. The student is thus reduced to a mere performer of tasks, or a passive listener or reader.

When we consider teaching engineers, one of the consequences of the teaching of mathematics based on information and on results alone is that students tend not to develop their intuitive thought, because their attention and activities are oriented by pre-established rules and laws. Besides, they tend not to develop enough self-confidence for evaluating the procedures used in the resolution of problems, or for trying different alternatives and new approaches. The result of such a form of behavior, induced by the current teaching models and learned by the student, is a tendency to passivity, which doesn't add to the development of a form of behavior that is creative and important for society.

As we approach the 21st century, our reality is constantly and rapidly changing. Part of these changes are mainly related to great advances of computer science. The automation of systems, for example, is changing the scenery of work places; integrated information is altering technological and social behavior. Situations are being modified, and what is taught in the context of Mathematics in higher education is not enough to enable apprentices to deal with problems that come up in a reality under constant change, in which they will act as professionals when they graduate. Longo [1] highlights the need of restructuring teaching in the courses of Engineering in order to prepare professionals who are able to work with the reality in order to achieve relevant results. This reality is influenced by different aspects, such as: a) fast changes (it is not enough "to master an occupation"); b) changes in technological, cultural and social paradigms in a time span shorter than past years; c) "real-time" information: it is neither enough nor appropriate to think just in "local terms"; d) science-based technology: to copy is self-defeating; e) techniques and algorithms are quickly overcome. These aspects, which are characteristic of the reality our students will deal with, require cognitive competence compatible with professionals with an ability to "learn how to learn", besides elaboration, creation and enterpreneurship as basic characteristics of a professional profile necessary and required in society today.

In order to meet current requirements, teaching must be conceived as process capable to modify the patterns of the students' behavior, enabling them to work with the situations they face, in such a way that they are able to achieve results that are considered relevant, according to Botomé [2]. Thus, the planning of a teaching program for a university course, for example, needs to take into account the situations the student will face as professional. It is then necessary that the teacher clearly and precisely establishes: a) what behaviors need to be taught in order to qualify the students to act performing changes that are considered important in face of contemporary needs of the social and technological reality; b) how to develop what was established as necessary to qualify students. Behavior here is understood as the relationship between that which the subject does and the environment in which he acts. (Skinner [3], [4], [5], [6], [7], [8], [9], Pavlov [10], Staddon [11], Pessoti [12], Sckick [13], Catania [14], [15], and Botomé [2], [16]).

In order to program teaching in such a way as to develop forms of behavior (understood as a relationship between what the subject does and the environment in which he acts), competencies and abilities that qualify students to evaluate processes, to criticize, to create, to interfere and to act according to situations, achieving relevant results, it is necessary to answer the following questions: a) how can teaching enable one to deal with real situations? ; b) what aspects characterize the reality the student will face in his daily routine as a professional?; c) which results interest us, in which context and to what extend?; d) how can the knowledge already produced help in this qualification process?; e) how is it possible to transform the knowledge in behavior, in the sense of "establishing a relation" with the environment through cognitive competencies, yielding the results required?

Botomé [16] examines different ways of understanding the concepts of "teaching," "learning," "teaching-learning process," and "teaching objectives", as he compares, deepens and expands the concepts given by several researchers, and points out controversies, conceptual misunderstandings, contradictions and new possibilities of understanding the concepts examined. Then Botomé [16] elaborates technology useful in the analysis and program of teaching, contributing for the construction of teaching programs, especially in higher education, that qualify individuals to establish relationships with the environment in which they live, achieving significant results.

Freire [17], [18], Ribeiro [19], [20] and Demo [21], [22] defend the idea that the main goal of teaching is to qualify people to work with reality so that they can take part of the technical, scientific and cultural development of their time. Considering these authors' ideas, Botomé [2] emphasizes that the main objective of higher education is to derive new possibilities of performance and different alternatives for solving problems, starting from the results of researches and literature available. In order to do so, besides developing cognitive competencies for dealing with situations from a technological point of view, higher education needs to develop cognitive competencies that characterize a person who is psychologically, emotionally, ethically and scientifically mature, capable of acting in a complex system of relationships.

From this point of view, learning is characterized by a change in the student's behavior, that is to say, the student's relationship with reality is modified, somehow and to some extent, as a consequence of teaching. In other words, teaching aims at modifying the students' patterns of behavior. The conception of behavior behind this idea of teaching implies not only action and performance, but a *relationship* between *action,* the *aspects of the situation* in which this action takes place, and the *consequences* and *modifications* produced in that specific situation because of the action performed. Such a conception of behavior, involving the relationship between action, situation and results produced, seems to be more convenient to help understand teaching as process that can transform knowledge into the ability of dealing with situations of the reality, as opposed to a conception of behavior merely as physical or verbal action, Botomé[16].

Considering higher education the kind of edication that qualifies people for a systematic performance and behavior a relationship between action and environment, Soares [23] proposes a new approach to the teaching of Mathematics in higher education so that it may develop in individuals abilities for an integrated performance, using mathematical resources to represent phenomena and to solve the problems they will face in their professional environment. Due to this role assigned to higher education, the author highlights that one of the important starting points for planning the teaching of Mathematics in higher education is the identification of situations professionals will face and need to be able to deal with, achieving results that may be considered relevant, based on mathematical knowledge. Thus, the teaching of Mathematics may be something concrete and contextualized in the students' academic formation, instead of being some discourse that fulfills academic and formal requirements, though many times meaningless outside the academic context.

Soares [23] offers some answers, although partial, for questions such as: What cognitive competencies (relationships with the environment) are necessary to teach Mathematics to prospective engineers? What situations does (or can) an engineer deal with (or needs to deal with)? What results need to be achieved? What actions (chain of actions) must be performed in order to achieve such results? And based on these data, how can one teach mathematical behaviors?

The analysis performed by Soares [23] is based on data and information obtained from interviews with mechanical engineers who work in large mechanic and metal-mechanic companies in the northeast region of the state of Rio Grande do Sul, in Brazil. The analysis and interpretation of the data aimed at finding significant mathematical competencies that could be taught to students of Engineering. The cognitive competencies found either are or can be the objectives of teaching Mathematics in higher education. From this point of view, the teaching of mathematics may then be conceived as a behavioral process, understood as a process that explicates the components involved in a form behavior, characterizing that which is done and its relationship with the situation before and after some action is taken. That behavioral process is defined by the relationships that are established between the students' context, the activities planned by the teachers and performed by the students, and the results of those activities and the way students relate with their reality. The results produced by the behavioral process of "teaching mathematics" concern the students' learning which is evaluated in this context in terms of the changes in their behavior after the teaching process. That is to say, the relationship between the student's actions and his/her environment (either academic or professional) is modified, which means that he/she is now able to work with this environment, "using" mathematical concepts differently from what he/she used to do before the learning process and achieving significant results. These results too are different from the ones that he/she was able to achieve before the learning process.

Seeing the teaching of mathematics in higher education as a behavioral process seems to qualify learners better to behave in a more systematic way, preparing them to act in a complex system of relationships, and developing their ability to deal with situations from a technological, scientific and professional point of view. The traditional concept of the teaching of mathematics is limited to the development of abilities related to algebraic manipulations, arithmetic manipulations, use of calculation rules and of techniques and formulas, often dissociated from its possible relationships with the environment.

A first organized set of the data obtained by Soares [23] enabled the elaboration of five groups of data: the first one regards activities performed by engineers that are related with mathematics; the second group regards general difficulties related to mathematics; the third one involves the mathematical concepts used by the engineers in their daily activities; the fourth group regards the concepts used, from a list of mathematical concepts taken from the syllabus of the engineering program; and the fifth one is related to the situations mentioned by the subjects interviewed as the ones in which they consider engineers need to use mathematical concepts.

The first group of data reveals that, according to the information from the subjects, the most frequent activities related to mathematics are (in order of frequency of indication): making calculations; using computer resources; designing parts, equipment and products; using mathematical tables, manuals and formulas; applying mathematics; optimizing methods; measuring parts; solving problems of mathematical nature; collecting data; drawing; identifying relationships between mathematical variables; identifying mathematical variables; interpreting results, graphs and tables; elaborating mathematical models or solving problems.

The second group of data shows that the difficulties related to mathematical knowledge, as indicated by the subjects, are (in order of frequency of indications): applying mathematical knowledge; solving problems; elaborating models and equating problems; interpreting results and problem situations; gathering data; remembering concepts; working with equations and formulas.

The third group of data reveals that the mathematical concepts used by engineers (in order of frequency of indication) are: trigonometry; basic mathematics; basic operations; calculations of structures and elements related to resistance of materials; plane and spatial geometry; lineal algebra; matrices and analytic geometry; differential and integral calculus of functions of a variable; 1st- and 2nd-degree algebraic equations; statistics and reliability; calculation of areas; perimeters and volumes; differential equations (formulas, ready solutions); analysis and interpretation of graphs; calculations related to the conversion and transformations of units; geometric drawing. That is to say, the mathematical concepts used are basically the concepts involved in the syllabuses of mathematics in junior high and high school.

The fourth group of data reveals that most of the concepts present in the syllabus of Mathematics for Engineering programs are either rarely used by engineers or not used at all. According to the information gathered, based on a list of mathematical concepts that are generally present in the syllabus of Engineering programs, the concepts more frequently used (in order of frequency of indication) are: trigonometry; spatial geometry; plane geometry; polar coordinates; curve plotting; statistics; probability; surface plotting; vectors in a plane; ordinary differential equations; integral of functions of a variable; logarithm; ordinary derivation; cylindrical coordinates; spherical coordinates; vectors in space. Concepts related to partial derivation, double integral, series of functions, series of potencies, numeric series, partial differential equations, vector functions, gradient field, line integrals, Laplace transformed, directional derivation, integral of surface and transformed of Fourier, which were all included in the list presented, were each mentioned either once, twice or not at all by the subjects interviewed.

The fifth group of data shows some of the situations, as indicated by the subjects, in which engineers need mathematical knowledge: calculation of resistance, efforts, tensions, structures; planning and analysis of projects; calculations of costs and budgets; calculations of figures related to drive, friction, speed, rotation; sizing and measuring; use of software (CNC, finite elements and others); calculation of area and volume; development of all activities related to engineering; use or application of equations, formulas, tables; analysis and resolution of problems, equating; development of projects; statistics and reliability; changes in scale; comparison of values; interpretation of drawings, graphs; laboratory, tests; research.

Summarizing, a preliminary analysis and interpretation of the groups of data elaborated by Soares [23] allows one to conclude: a) activities performed by engineers in their workplace are related to the “use” of rules, formulas, equations and definitions, referring, most of the time, to basic mathematics concepts; b) the most frequent difficulties related to mathematics which engineers face in routine activities concern the use of rules, formulas, equations and definitions, which are mathematical resources; c) the situations which engineers face in their professional environment, and in which they need mathematics, are related to the use of the mathematical resources. Information from the subjects thus reveal that, in terms of mathematics, the difficulties and situations they face are related to “applying mathematics.” In other words, in terms of mathematics, what they most often do and their greatest difficulties are both related to “applying mathematics.”

Thus, one can say that “applying mathematics” is an expression that synthesizes the cognitive competencies that need to be taught to engineers in order to qualify them to work with their environment, starting from mathematical knowledge, producing relevant results. The idea of “applying” a mathematical concept is related to “using” the concept in order to perceive, identify and analyze a phenomenon, in order to obtain data, to solve a problem, to calculate a numeric value, to take measurements, to develop projects, to compare values, to interpret graphs, to draw, to research something, to equate, to process data or to optimize processes.

How can one understand the expression “applying mathematics” as the relationships that may be established between a given situation context, mathematical knowledge, actions performed and results obtained by means of those actions? Answering this question takes one to the understanding of the expression “applying mathematics” as mathematical behavior, that is, as “transforming mathematical knowledge in relevant behavior”. Understanding the teaching of Mathematics in higher education as this, the relationships established among the situation context, the actions performed, the results obtained and the mathematical concepts are the components of the new behavior to be formed (or synthesized, according to Botomé, [16]) by means of teaching. Thus, “transforming mathematical knowledge into mathematical behavior” is a more appropriate expression than “applying mathematics” to refer to activities performed by engineers, because “transforming mathematical knowledge into mathematical behavior” expresses better what the process which the expression “applying mathematics” refers to consists of.

The interpretation of the data obtained by Soares [23] made the characterization of situations mechanical engineers face in their professional life possible. This is just a starting point to derive, describe, characterize, systematize, and decompose mathematical forms of behavior identified as the ones that mechanical engineers need to develop in order to be able to deal with their professional environment, achieving significant results. These forms of behavior are part of the group of important cognitive competencies, in terms of mathematics, that engineers need to show in order to successfully (and with quality) manage daily routines at work. Identifying and teaching these cognitive competencies involves changing concepts and habits of the teachers of mathematics to engineers.

Teaching mathematics requires the development of a specific technology for teaching engineers by means of the creation of “teaching conditions" supported on objectives that are significant competencies for the exercise of the profession in contexts where engineers may work, as represented by the forms of behavior established from the results of the interpretation of the data presented. Teaching mathematics is a form of enabling individuals to relate with their professional environment, taking into account mathematical concepts and producing important results from a professional, social and scientific point of view. The discovery of such cognitive competencies (the abilities aimed at) and their processing so that they can be learned is what is being summarized under the name of behavioral technology in higher education, Botomé [24].

In other words, the characterization and systematization of mathematical cognitive competencies (behaviors, in the context of mathematics) are the basis of the planning of "teaching conditions" by means of which learners can establish relationships between the activities they perform, situations from their environment and results that need to be achieved by means of such activities. Thus, the next stages for the construction of knowledge that enables planning teaching syllabuses, in this sense, are: a) derive important mathematical cognitive competencies that will be chosen as the objectives of the teaching of mathematics in higher education for engineers; b) identify and describe the components of each desired form of behavior; c) decompose the desired cognitive competencies into intermediary learning processes taking into account the components described in each one of them (second stage of decomposing of objectives); d) establish a sequence and organize the "intermediary learning processes" in learning chains and in performing chains; e) organize the learning chains in groups that constitute units of teaching of mathematics for engineers. Such a process can allow for the construction of a basis on which to develop mathematics syllabuses that may qualify engineers to deal with their professional environment, using mathematics and producing results that are considered important in face of the demands of the present time. The next stage is "applying" the teaching syllabuses and evaluating the efficiency and effectiveness of this application (Botomé, [16], [24] and Soares [23]).

The planning of the teaching of mathematics in higher education for engineering programs is generally based on the curricula, on "contents" of the courses. The teacher plans "teaching conditions" with emphasis on activities, such as exercises and papers, accomplished after the presentation (information) about "the contents." The student performs the tasks asked by the teacher and there's a (false?) sense of duty discharged because that which is routine and usual has been done, the actions proposed were taken.

But is it still possible to keep this question in mind: what do these actions allow "to learn?" If learning is defined as the apprentice's training of the ability to work with the situations he/she will face (either in the academic or the professional context), then it is not hard to suspect that not much or almost nothing has been learned. Examination of the data and information supplied by the subjects allows us to infer that the difficulties related to mathematics found by engineers in their daily activities are partly due to a way of teaching that, in general, doesn't qualify learner to relate concepts with situations and with the results that need to be produced by actions that are related to these concepts. If the planning of the teaching of mathematics for engineers continues centered on the "contents" proposed by the academic programs, without taking into account the environment in which the prospective professionals will work, it is unlikely that higher education planners teach competencies that are significant for those professionals' performance in their work environment starting from the mathematical knowledge.

Deriving important professional cognitive competencies from the analysis and interpretation of information gathered from subjects that work in the normal professional routines for which one wishes to prepare individuals to perform in (achieving significant results) seems to be an useful instrument to plan teaching that results in the development of abilities of learners. That is to say, proposing objectives for the teaching of mathematics for engineers having situations engineers face (or may face) as a starting point, deducing results to be produced and actions that allow such results to be achieved, "using" mathematical knowledge, is fundamental to enable a form of teaching that may have as a consequence the learner's ability to deal with situations of his/her professional daily routine.

Such a way of planning of the teaching of mathematics in higher education for engineers does not ensure by itself that alumni achieve significant results through their behavior. However it is possible to infer that this way of planning teaching increases the student's probability of transforming the information on mathematics into competencies which will result in significant product in their professional environment.

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