Comparison Between Theoretical Models and Experimental Measurements of Electrodinamic Loudspeaker Systems.

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Abstract- *The models based in dynamical analogies are very easy to simulate with the aid of computer numerical software. The main advantage of these models is that they work well if no high precission is required. In this paper we describe the process of a loudspeaker system modelling, the computer simulation and the comparison with experimental results.*

Introduction

In this paper we describe the use of equivalent circuits based on dynamical analogies and the computer simulation of the electrodymanic loudspeakers in Electroacoustics teaching. This is a normal procedure we have developed during the last three year in different subjects in our curriculum of Telecommunication Engineering. In the last years, we have tried to introduce the use of computer similation in Electroacoustics, because its complexity and because the need of physical and mathematical knowledge for the students. The simplest approximation we have found to study the behaviour of electrodynamical loudspeakers is the use of electrical equivalent circuits based on dynamical analogies [1],[2]. With these analogies, we can made models to study complicate transducers with no high precission and with the later comparison with experimental results we could verify their validity. These circuits are easy to analyze by the students with the tools they have got in previous subjects, but it is possible that, for their simplicity, they can modelate completely the transducer, specially in the radiation parts.

We will describe an application developed with a commertial numerical calculation program of a three ways electrodynamic loudspeaker system mounted in a closed acoustical box and with a passive crossover network. The way we use with the students is to propose them an equivalent circuit they have to analyze and simulate with the aid of a numerical program. In this case we propose they to use Mathcad[1] [3] to make the simulation. Normally, we use commertial programs to make simulations during the subjects. Also, we propose them to create their own simulation in C or C++, we these languages need programming skills that have not all the students. Mathcad is a very powerful calculus program that not need high programming and has a great graphic possibilities that allows the user not to program all the simulation elements.

Equivalent circuits

The transducer system we are going to simulate is closed box with an internal enclosure of 158 liters, with three loudspeakers (a 15” woofer, a 8” mid-range and a 2” tweeter) and a passive crossover network. The first step, is to create the model with the aid of equivalent circuits. We propose to use a lumped element circuit for each loudspeaker. The equivalent circuit is shown in figure1.

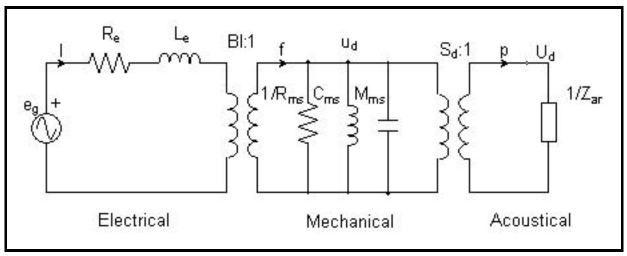

Figure 1. Equivalent circuit of an electrodynamic loudspeaker, showing the three parts: electrical, mechanical and acoustical.

As we can see in figure 1, the circuit is very simple to understand an to analyze. The two ideal transformers represent the energy transduction beetwen the electrical and mechanical part (Bl:1, or force factor) and beetwen mechanical and acoustical part (S_d:1, efective radiation surface). In the electrical part we have the representation of the coil autoinduction (R_e and L_e), in the mechanical part the mechanical elements of the diaphragm (R_{ms}, M_{ms} and C_{ms}) and in the acoustical part we have the representation of the diapragm radiation (Z_{ar}).

In our simulation is very convenient to use another equivalent circuit, all in the acoustical part, because we will try to find the pressure response of the system. This equivalent circuit appears in figure 2.

In this circuit, we can see three different banches, with their corresponding volume velocities, U_d in the diaphargm branch, U_b in the inner box enclosure branch

[1] Mathcad is a registered product of MathSotf Inc.

and U_l in the leackage losses branch. The equivalent genator p_g will be different for each loudspeakers depending on the crossover network.

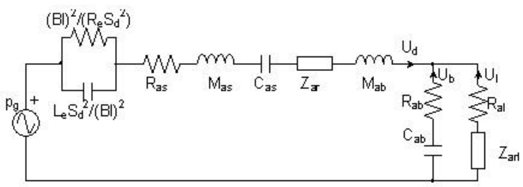

Figure 2. Acoustical equivalent circuit of a loudspeaker, mounted in a closed box.

System Simulation and Measurement

Now we can begin with the simulation. The first step in introducing the values of each element of the equivalent circuit. We make this procedure in Mathcad, and we include some parts of the complete Mathcad application for space reasons.

* Woofer Small parameters:

$f_{sw} := 37$ Hz $\quad M_{msw} := 0.089$ kg $\quad Q_{msw} := 6 \quad S_{dw} := 0.08$ m²

$Bl_w := 17.2$ N/A $\quad R_{ew} := 6.3\ \Omega \quad L_{ew} := 1.3\cdot 10^{-3}$ H

* Mid-range Small parameters:

$f_{sm} := 88$ Hz $\quad M_{msm} := 0.016$ kg $\quad Q_{msm} := 2.84 \quad S_{dm} := 0.021$ m²

$Bl_m := 7.98$ N/A $\quad R_{em} := 5.6\ \Omega \quad L_{em} := 0.45\cdot 10^{-3}$ H

* Tweeter Small parameters:

$f_{st} := 1110$ Hz $\quad M_{mst} := 0.003$ kg $\quad Q_{mst} := 4.59 \quad S_{dt} := 0.0008$ m²

$Bl_t := 6.10$ N/A $\quad R_{et} := 5.2\ \Omega \quad L_{et} := 0.17\cdot 10^{-3}$ H

* Definition of geometrical parameters of the driver diaphragms (a_{dw}, a_{dm}, a_{dt}) volume enclosure (V_b) and the enclosure losses due to leackage (a_l, S_l):

$$a_{dw} := \sqrt{\frac{S_{dw}}{\pi}}\ \text{m} \quad a_{dm} := \sqrt{\frac{S_{dm}}{\pi}}\ \text{m} \quad a_{dt} := \sqrt{\frac{S_{dt}}{\pi}}\ \text{m}$$

$V_b := 0.158$ m³ $\quad a_l := 0.00001$ m $\quad S_l := \pi\cdot a_l^2$ m²

***Note**: in our model we supose the enclosure losses, in radiation terms, like a plane piston of radius a_l mounted in an infinite baffle.*

* Definition of the acoustical enclosure and losses elements:

$R_{ab} := 100$ N.s/m⁵ $\quad M_{ab} := 0$ kg/m⁴ $\quad R_{al} := 6\cdot 10^5$ N.s/m⁵

Now, we will define the acoustical elements of each branch, including radiation terms:

* Directivity factor of a plane piston of radius *a* mounted in an infinite baffle.

$$Q_{ax}(s,a) := \frac{\left(\frac{s}{j\cdot c}\cdot a\right)^2}{1-\frac{J1\left(2\cdot\frac{s}{j\cdot c}\cdot a\right)}{\left(\frac{s}{j\cdot c}\cdot a\right)}}$$

* Complete acoustical elements of the radiation for a plane piston of radius a mounted in an infinite baffle:

$$R_{ar}(s,a) := \frac{\rho_0\cdot c}{\pi\cdot a^2}\cdot\left(1-\frac{J1\left(2\cdot\frac{s\cdot a}{j\cdot c}\right)}{\frac{s\cdot a}{j\cdot c}}\right) \quad X_{ar}(s,a) := \frac{\rho_0\cdot c}{\pi\cdot a^2}\cdot\frac{H1\left(2\cdot\frac{s\cdot a}{j\cdot c}\right)}{\frac{s\cdot a}{j\cdot c}}$$

***Note**: J1 is the 1st order Bessel function and 1st order Struve function.*

***Note**: in our model we supose all elements radiate like a plane circular piston. Other supossitions may be assumed. The user could use asynthotical aproximations to these functions.*

In this part of Mathcad application, we can see that we modelate the radiation of each driver like a circular piston mounted in an infinite baffle, where J1 is the first oreder Bessel function and H1 is the first order Struve function. This is our supposition, because simulate more exactly the radiation behaviour of the diaphragm would be very complicate for an application like this for our students. With other simulation programs, more specific for electroacoustics transducers, we could evaluate the radiation better. Also, we use the complex variable s (s=jω) to analyze the frucuency response.

* Drivers acoustical elements:

$$M_{asw} := \frac{M_{msw}}{S_{dw}^2} \quad C_{asw} := \frac{S_{dw}^2}{M_{msw}\cdot(2\cdot\pi\cdot f_{sw})^2} \quad R_{asw} := \frac{M_{asw}\cdot 2\cdot\pi\cdot f_{sw}}{Q_{msw}}$$

$$M_{asm} := \frac{M_{msm}}{S_{dm}^2} \quad C_{asm} := \frac{S_{dm}^2}{M_{msm}\cdot(2\cdot\pi\cdot f_{sm})^2} \quad R_{asm} := \frac{M_{asm}\cdot 2\cdot\pi\cdot f_{sm}}{Q_{msm}}$$

$$M_{ast} := \frac{M_{mst}}{S_{dt}^2} \quad C_{ast} := \frac{S_{dt}^2}{M_{mst}\cdot(2\cdot\pi\cdot f_{st})^2} \quad R_{ast} := \frac{M_{ast}\cdot 2\cdot\pi\cdot f_{st}}{Q_{mst}}$$

* Drivers electrical impdance:

$$Z_{ew}(s) := R_{ew} + s\cdot L_{ew} \quad Z_{em}(s) := R_{em} + s\cdot L_{em} \quad Z_{et}(s) := R_{et} + s\cdot L_{et}$$

* Enclosure acoustical compliance (assuming no filling material inside):

$$C_{ab} := \frac{V_b}{1.4\cdot 10^5}$$

* Equivalent drivers acoustical impedance:

$$Z_{aew}(s) := \frac{Bl_w^2}{Z_{ew}(s)\cdot S_{dw}^2} \quad Z_{aem}(s) := \frac{Bl_m^2}{Z_{em}(s)\cdot S_{dm}^2} \quad Z_{aet}(s) := \frac{Bl_t^2}{Z_{et}(s)\cdot S_{dt}^2}$$

* Radiation acoustical impedance:

$$Z_{ar}(s,a) := R_{ar}(s,a) + j\cdot X_{ar}(s,a)$$

* Diaphragms brach acoustical impedance, including the equivalent driver acoustical impedance

$$Z_{aswt}(s) := \left(R_{asw} + \frac{1}{s\cdot C_{asw}} + s\cdot M_{asw}\right) + Z_{ar}(s, a_{dw}) + Z_{aew}(s)$$

$$Z_{asmt}(s) := \left(R_{asm} + \frac{1}{s\cdot C_{asm}} + s\cdot M_{asm}\right) + Z_{ar}(s, a_{dm}) + Z_{aem}(s)$$

$$Z_{astt}(s) := \left(R_{ast} + \frac{1}{s\cdot C_{ast}} + s\cdot M_{ast}\right) + Z_{ar}(s, a_{dt}) + Z_{aet}(s)$$

The next step is introducing the crossover network in the system. The passive croosver is shown in figure 3.

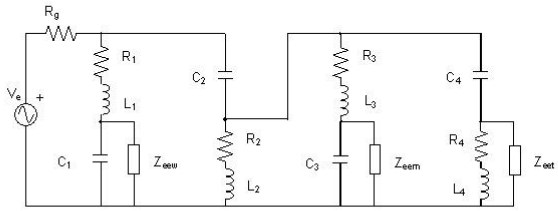

Figure 3. Three ways passive crossover network. Z_{eew}, Z_{eem} and Z_{eet} represent electrical charge of each driver.

The value of the crossover network are:

$L_1 \equiv 5\cdot 10^{-3}$ H	$R_1 \equiv 2.7$ Ω	$C_1 \equiv 10\cdot 10^{-6}$ F
$L_2 \equiv 1.3\cdot 10^{-3}$ H	$R_2 \equiv 1$ Ω	$C_2 \equiv 10\cdot 10^{-6}$ F
$L_3 \equiv 0.5\cdot 10^{-3}$ H	$R_3 \equiv 0.5$ Ω	$C_3 \equiv 6.8\cdot 10^{-6}$ F
$L_4 \equiv 0.33\cdot 10^{-3}$ H	$R_4 \equiv 0.5$ Ω	$C_4 \equiv 3.3\cdot 10^{-6}$ F

The electrical charge of each driver can be represented by the next expressions and the simulated values are shown in figure 4:

Driver voice-coil impedance calculus:

$$Z_{asw}(s) := \left(R_{asw} + \frac{1}{s\cdot C_{asw}} + s\cdot M_{asw}\right) + Z_{ar}(s, a_{dw})$$

$$Z_{asm}(s) := \left(R_{asm} + \frac{1}{s\cdot C_{asm}} + s\cdot M_{asm}\right) + Z_{ar}(s, a_{dm})$$

$$Z_{ast}(s) := \left(R_{ast} + \frac{1}{s\cdot C_{ast}} + s\cdot M_{ast}\right) + Z_{ar}(s, a_{dt})$$

$$Z_{eel}(s) := Z_{ew}(s) + \frac{Bl_w^2}{S_{dw}^2}\cdot\left[Z_{asw}(s) + \left(Z_{ab}(s)^{-1} + Z_{al}(s)^{-1}\right)^{-1}\right]^{-1}$$

$$Z_{eem}(s) := Z_{em}(s) + \frac{Bl_m^2}{S_{dm}^2}\cdot\left[Z_{asm}(s) + \left(Z_{ab}(s)^{-1} + Z_{al}(s)^{-1}\right)^{-1}\right]^{-1}$$

$$Z_{eeh}(s) := Z_{et}(s) + \frac{Bl_t^2}{S_{dt}^2}\cdot\left[Z_{ast}(s) + \left(Z_{ab}(s)^{-1} + Z_{al}(s)^{-1}\right)^{-1}\right]^{-1}$$

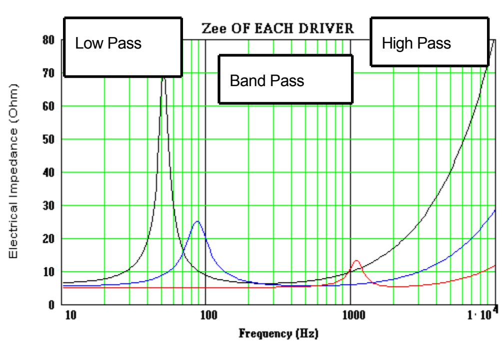

Figure 4. Simulated values for Z_{eew}, Z_{eem} and Z_{eet}
Z_{eew} (black line) Z_{eem} (blue line) Z_{eet} (red line

The next step is the analysis of the crossover network to find the equivalent pressure for each driver. We propose to use the current method to find the equivalent voltage we have to introduce in the generator of figure 2 for each driver. The equivalent voltage in each driver apperars in figure 5.

To make the comparison with experimental results we will use an electroacoustic measurement system based on maximum length sequences signals (MLS) [4, MLSSA[2]] [5]. With this measurement system we can get the transfer function of the three ways crossover network as we see in figure 6. As we can see the simulated and the measured transfer function of each filter match well, except for the high pass filter. When the student notice this problem would have to fit the model in some parameters until to get the correct simulated response. All the measurements were made in the same conditions than the model, using like charge in each filter output the corresponding loudspeaker mounted in the box.

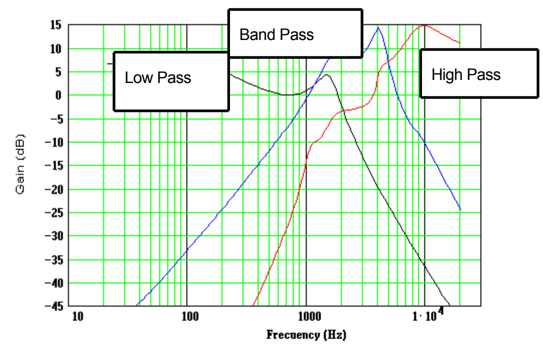

Figure 5. Simulated transfer function of the three ways crossover network.
LPF (Black line) BPF (Blue line) HPF (Red line)

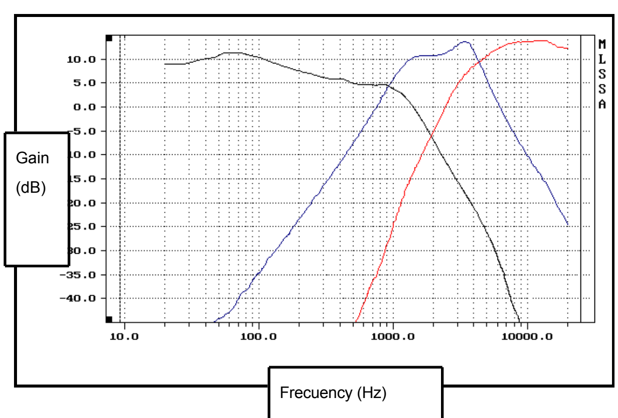

Figure 6. Measured transfer function of the three ways crossover network.
LPF (Black line)BPF (Blue line) HPF (Red line)

After this, we have to calculate the equivalent current for each branch of the equivalent circuit of figure 2. These are the volume velocities (m^3/s) in the surface of each

[2] MLSSA is a registered product of DRA Labs.

diaphragm. Next, we will calculate the radiated pressure from each driver. To do this, we suppose that the pressure in the diaphragm surface in the product of the corresponding volume velocity by the complex radiation impedance. When we have to calculate the radiated pressure in sensitivity conditions (on axis, 1m, 1W), we suppose spherical radiation from the center of each driver (we applicate spherical divergence), like if each one radiate like an sphere of the corresponding radius. This supposition works well if we compare it with the low frecuency Small analysis [6],[7].

* Equivalent pressure generator:

$$p_{gw_i} := (V_{l_i}) \cdot \frac{Bl_w}{Z_{ew_i} \cdot S_{dw}} \quad p_{gm_i} := (V_{m_i}) \cdot \frac{Bl_m}{Z_{em_i} \cdot S_{dm}} \quad p_{gt_i} := (V_{h_i}) \cdot \frac{Bl_t}{Z_{et_i} \cdot S_{dt}}$$

* Volume velocity of the woofer diaphragm branch:

$$U_{dw_i} := p_{gw_i} \cdot \left[Z_{aswt_i} + \left[(Z_{ab_i})^{-1} + (Z_{al_i})^{-1} \right]^{-1} \right]^{-1}$$

* Volume velocity of the losses branch: * Total output volume velocity:

$$U_{lw_i} := -U_{dw_i} \cdot \frac{Z_{ab_i}}{Z_{ab_i} + Z_{al_i}} \qquad U_{tw_i} := U_{dw_i} + U_{lw_i}$$

* Radiated pressure from the woofer diaphragm: * Radiated pressure from the losses:

$$p_{dw_i} := U_{dw_i} \cdot Z_{arw_i} \cdot \left(\sqrt{\frac{S_{dw}}{4 \cdot \pi}} \right) \qquad p_{lw_i} := U_{lw_i} \cdot Z_{arl_i} \cdot \left(\sqrt{\frac{S_l}{4 \cdot \pi}} \right)$$

* Total radiated pressure:

$$p_{tw_i} := p_{dw_i} \cdot \sqrt{Q_{axw_i}} \qquad SPL_{w_i} := 20 \cdot \log \left(\frac{p_{tw_i}}{2 \cdot 10^{-5}} \right)$$

The corresponding radiated pressures from the other drivers have similar expressions that above.

Finally we have to evaluate the total radiated pressure from the box system. To make this we evaluate the individual radiated power of each driver like we shown before for the woofer (bass unit) and finally we make a complex addition of the three radiated pressure. In this addition we do not take account the relative position between the loudspeakers in the box, because we suppose that the measurements are made in far filed conditions. All the measurements were made in an anechoic chamber on the box axis, at 1 meter long and with an electrical excitation of 1 W with MLSSA. The results of the simulation and measurement are shown in figures 7 and 8, respectively.

* Total radiated pressure from the woofer diaphragm:

$$p_{tw_i} := p_{dw_i} \cdot \sqrt{Q_{axw_i}} + p_{lw_i} \cdot \sqrt{Q_{axl_i}}$$

* Total radiated pressure from the mid-range diaphragm:

$$p_{tm_i} := p_{dm_i} \cdot \sqrt{Q_{axm_i}} + p_{lw_i} \cdot \sqrt{Q_{axl_i}}$$

* Total radiated pressure from the tweeter diaphragm:

$$p_{tt_i} := p_{dt_i} \cdot \sqrt{Q_{axt_i}}$$

* Total total radiated pressure:

$$p_{T_i} := p_{tw_i} + p_{tm_i} + p_{tt_i} \qquad SPL_{T_i} := 20 \cdot \log \left(\frac{p_{T_i}}{2 \cdot 10^{-5}} \right)$$

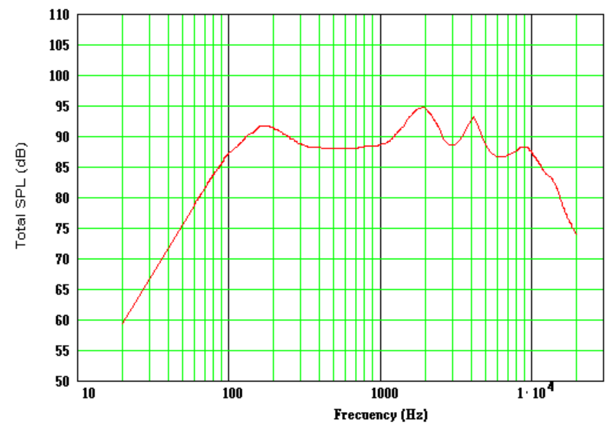

Figure 7. Simulated total radiated sound pressure level with complex addition of each driver.

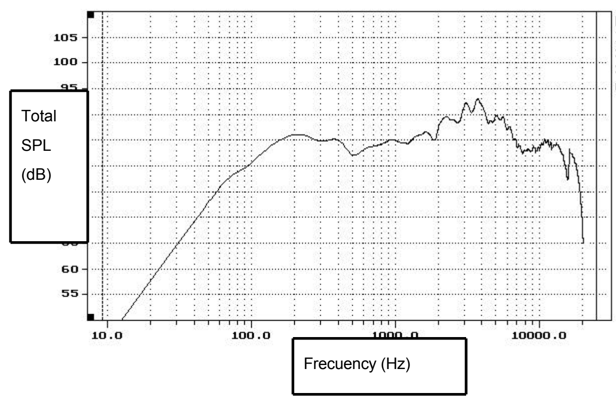

Figure 7. Measured total radiated sound pressure level

Conclusions

We have use this multiple technique of using equivalent circuits to make a model of loudspeakers behaviour, simulate the response of them with the use of numerical calculus programs and extract the experimental results from laboratory measurements with low cost equipment during the last three years. By one hand, the use of equivalent circuits allows the student developing analysis techniques they have got before in our career and so see the could link a practical approach of the classical circuit analysis. We could complicate these lumped elements equivalent circuits to model the low frecuency of the loudspekers system. As we see, the high frecuency response require a more complicate model based on transmission lines or finite elements approxomation, but these method need more knowledges for our students. The use of computer simulation reduce the calculus time and allows the students analyze different cases with no high programming skills. We usually use Matcad or Matlab in the normal time of class. The joining of experimental results with the simulated ones brings near the students to the reality of complicate phenomena ralated with transducers and allows validate the models they have developed.

In our experience in this years, we have notice that the results of students curricula have improved with the use of the computer during the timetable, not only in the practical credits but in the theoretical

References

[1] Olson H.F., *Acoustical Enginnering*, Van Nostrand, New York, 1.957.

[2] Beranek L.L., *Acoustics*, McGraw Hill, New Jersey, 1.954.

[3] Mathcad 6.0 PLUS User´s Guide, MathSoft Inc., Massachusetts, 1.995.

[4] Rife D., Vanderkooy J., “Transfer-Function Measurements with Maximum Length Sequences”, Journal of the Audio Engineering Society, Vol. 37, Nº 6, June 1.989.

[5] MLSSA 10.0A User´s Guide, DRA Labs. Florida, 1.996.

[6] Small R.H., “Direct Radiator Loudspeaker System Analysis”, in *Loudspeakers: an anthology* Vol 1, Audio Enginnering Society, New York, 1.980.

[7] Gomez J.J., Recuero M., Sanchez J.L., “The use of computer simulations in Electtroacoustics Transducer Teaching”, Proceedings of the 100th AES Convention, Copenhagen, 1.996.