The Importance Of Showing Transfer Functions In Three Dimensions

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Abstract - *Numerous publications in the area of Electrical Engineering are concerned with Signals and Systems, Control Systems, Signal Processing and other subjects that are based on the Laplace and z transforms. As the variables of both transforms are complex, any function that depends on these variables may be represented in three dimensions (3D). In technical literature, however, most authors use only two dimensions (2D). For this reason students may have more difficulty when interpreting calculations, which may decrease their motivation. In order to make interpretation easier, 3D representations of transfer functions are being used in such disciplines at CEFET-PR, in both undergraduate and graduate levels, with better results being obtained. The aim of the present paper is to show the importance of giving students a chance of seeing and interpreting such functions in 3D.*

Introduction

A few decades ago, when slide rules were the most useful tool to make calculations, it was only possible for a few famous companies, research centers, space agencies and universities in first-world countries to make use of computers in areas such as business, engineering, research programs and education. Later, scientific calculators appeared in the market followed by home computers, which showed low-resolution graphics and were relatively expensive. Using them, it took half an hour or more, instead of a few seconds, to project on the screen a "Mexican hat" similar to the one shown in figure 1.

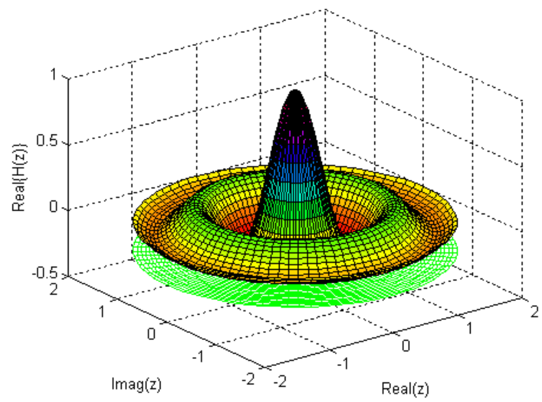

Figure 1. "Mexican hat".

Unfortunately, this change also brought problems that didn't exist before. Bad articulation of fingers and eye damage for those who use PC's every day and children who prefer to play with a computer instead of making friends, are just a few examples.

Projections in Three Dimensions

An artist may represent an object in the form of a drawing or a sculpture. Both forms have advantages and disadvantages. If it is to be used to decorate a wall, for example, a painting must be more interesting, but, if it must be appreciated from several directions, a sculpture will be much more advantageous than one or more drawings representing the object. This is the case, for example, for a globe compared with a cartographic map, which normally produces some distortion, or the representation of the solar system in three instead of two dimensions.

Since the human being has the capacity of seeing objects in three dimensions, it is natural to try to bring a little more of reality using stereoscopy to observe a drawing or to watch a cartoon, for example. Astronomers have used a series of pictures taken by the Hubble telescope showing different sides of planets to obtain the effect of their rotation in 3D.

Using software related to virtual reality, areas such as Architecture, Civil Engineering and Chemistry, to give just a few examples, make it possible to "go into the building" to inspect how it will look after being decorated or to see the structure of a complex molecule rotating in space. Thanks to multimedia, a child "goes to the zoo" and sees a lion walking and roaring, which, in some aspects, may be more interesting than taking a look at a book to see a picture of a lion. Even the reproduction of a sculpture is already possible with signals that correspond to the original object being generated by a laser ray at one end and then being transmitted just like a fax to the other end, where another laser ray reproduces a copy.

No matter how popular the area of Informatics become though, books are still the most powerful tools to transmit knowledge to new generations. So, textbooks must contain the necessary information, illustrations and examples, particularly when concerned with subjects that involve calculus, for example, in order to avoid too much abstraction. This will help the reader to appreciate the usefulness of the new concepts. In many cases, instead of a lot of words and equations, illustrations are the best way to transmit the main idea. For example, it is evident that equations 1 and 2 are not so attractive as their graphical representations shown in figures 2-(a) and 2-(b) respectively.

$$X(z) = \left(1 + z^2 + z^3\right)^{\frac{1}{3}} \quad \text{where}$$

$$z = a + jb \qquad j = \sqrt{-1} \tag{1}$$

$$Y(z) = z\frac{\sin\left(2\pi|z|\right)}{2\pi|z|} \tag{2}$$

The amplitude is proportional to the real values while the change of color, or contrast if printed in black and white, depends on the imaginary values

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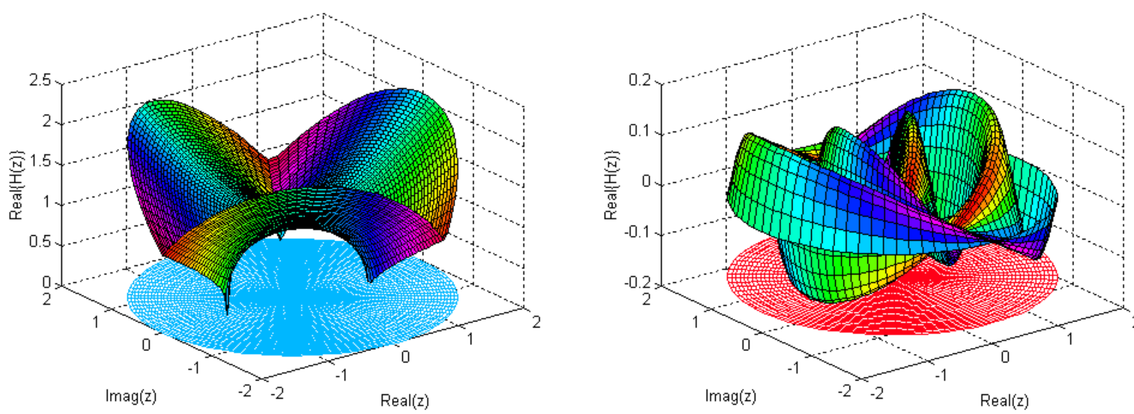


(a) (b)

Figure 2. Graphical representation in 3D of functions (a) X(z) and (b) Y(z).

Laplace, z and Fourier Transforms

Both the Laplace and the z transforms, which are based on complex variables (variables that have a real and an imaginary part that form a plane), can help to solve problems that involve complicated differential equations with the use of simple algebraic equations. The Laplace transform, defined by equation 3 and used in the analysis of analog systems, is based on the variable s=σ+jω while the z transform, defined by equation 4 and used in the analysis of digital systems, is based on the variable $z=e^{(j\theta)}$, where "j" represents the imaginary unit. The Fourier transform, defined by equation 5, is a special case of the Laplace transform when σ=0.

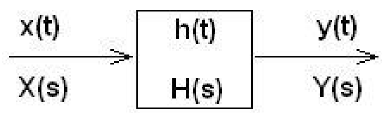
$$X(s) = \int_{0^-}^{\infty} x(t)\, e^{-st}\, dt \tag{3}$$

$$X(z) = \sum_{k=0}^{\infty} x(k)\, z^{-k} \tag{4}$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \tag{5}$$

Transfer Functions

A transfer function represents the modification in the output imposed by a system on its input [4, 10]. Represented by equation 6 for an analog linear system, it is the Laplace transform of the impulse response h(t) and is defined as the relation between the Laplace transforms of the output {y(t)} and input {x(t)} signals, as shown in figure 3. In case of a digital linear system, the z transform is used. The roots of the numerator of a transfer function are known as zeros while the roots of the denominator are known as poles.



$$Y(s) = X(s) \cdot H(s)$$

Figure 3. Transfer function of an analog linear system.

$$H(s) = \frac{Y(s)}{X(s)} \qquad (6)$$

Based on the positions the poles and zeros occupy on the s or z planes, many aspects of the system such as stability, frequency response and time response may be analyzed using algebraic equations. To investigate the implications caused by variations of poles and zeros, four basic types of two-dimensional (2D) graphics are normally used. They are: 1) the original positions of poles (represented by) and zeros (represented by **o**) on the complex plane; 2) the root locus showing possible changes of positions suffered by poles and zeros due to some feedback; 3) amplitude and phase responses as a function of frequency and 4) impulse and unit step responses as a function of time. As an example, such graphics are shown in figure 4 for the transfer function represented by equation 7.

$$H(s) = \frac{\sqrt{2}\, s}{s^2 + \sqrt{2}\, s + 1} \qquad (7)$$

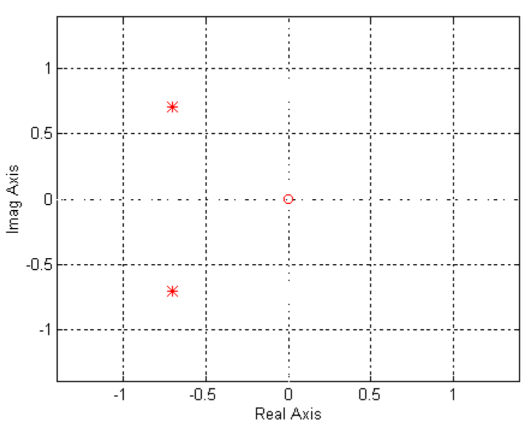
Studying and Understanding

The students normally consider subjects that depend on the Laplace, Fourier and z transforms as very difficult. If the teacher doesn't take care, the students will make calculations almost automatically, without knowing what is really going on. Among these subjects are Signals and Systems [4, 6, 7, 8, 9], Communication Systems [2, 14], Filters [1, 5, 17], Control Systems [10] and Digital Signal Processing [3, 11, 12, 13, 15, 16].

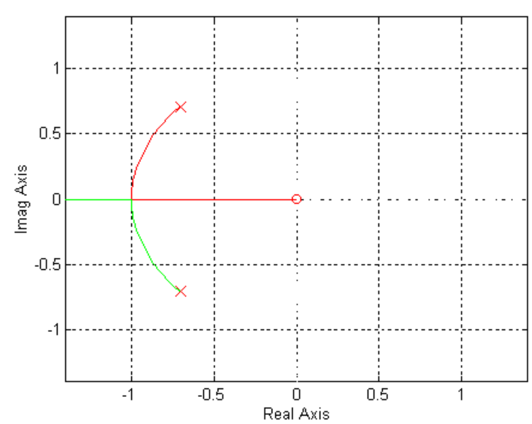
The theory of filters, for example, is dominated by such transforms. Active or passive analog filters and digital filters may be calculated directly on the basis of transfer functions that are formed by polynomials, some of which are listed in tables [4, 5, 15, 16]. Frequency responses may be obtained using specific equations or by transforming H(s) to H(jω) or H(z) to $H(z=e^{j\theta})$. Nowadays, software can solve this kind of problem very quickly but it is necessary to understand the obtained results.

Even if the students don't demonstrate some curiosity, they will certainly ask questions like: 1) If the variable has two dimensions, what will the aspect of the transfer function be in space when changes are made on the polynomial? 2) Why does the frequency response change so much just because a zero was included at s=0?

As books concerned with Analytic Geometry normally contain representations of objects in 3D, he/she may have already studied the equations and aspects of a sphere or an ellipsoid for example, but not necessarily those of a function that tends to zero and infinity at different points. Maybe the adopted textbook or another one indicated by the teacher contains at least one representation of a transfer function in a three-dimensional form [6, 9, 12, 14], making it easier for the students to understand its significance. As illustrated in figure 4, the main problem is that most authors don't include a perspective of a transfer function.



(a)



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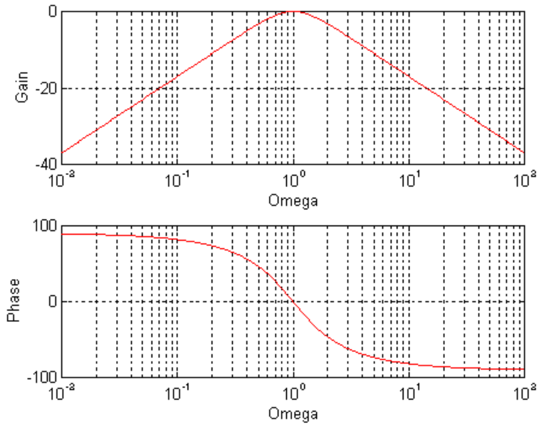

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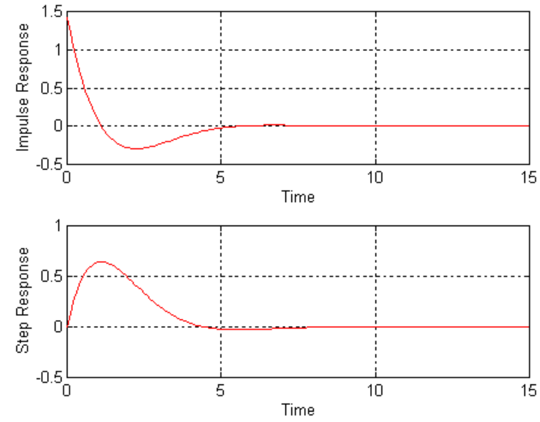
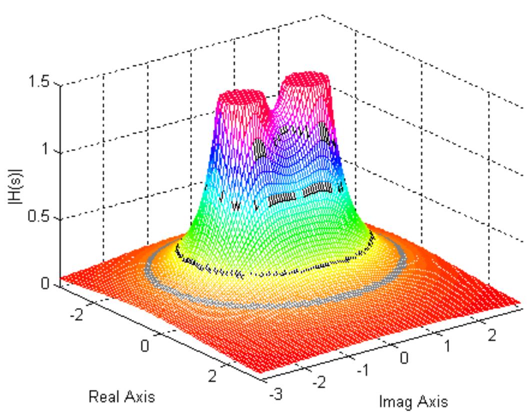


(d)

Figure 4. Usual graphical representations of H(s). (a) Original positions of poles and zeros, (b) *root locus*, (c) frequency and phase responses and (d) impulse and step responses.



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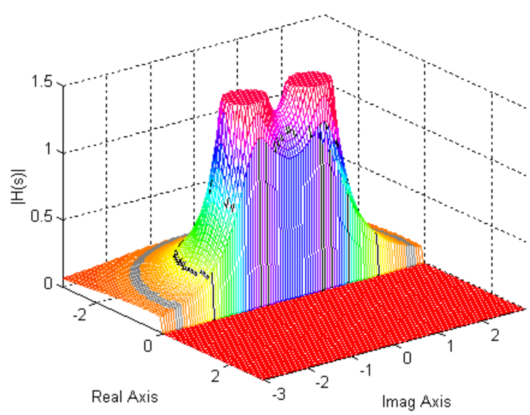
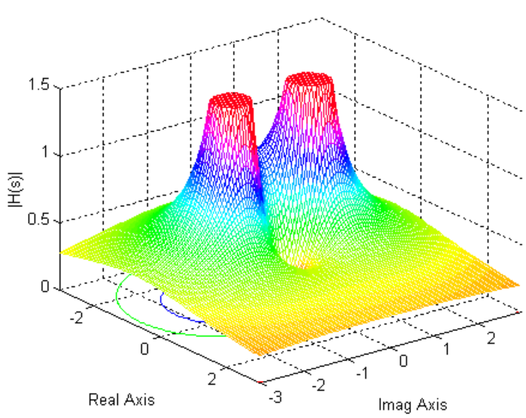
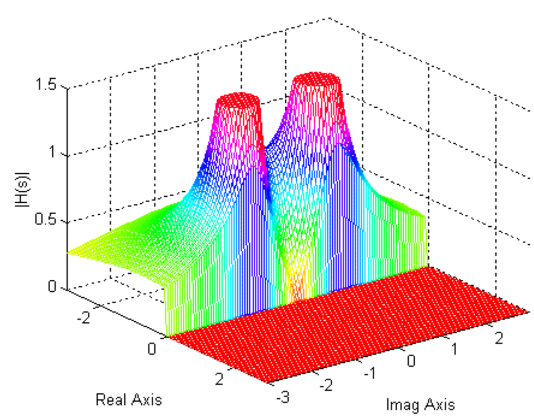


(b)

Figure 5. Absolute value of the transfer function of a second-order lowpass filter represented (a) over the *s* plane and (b) on the left-hand side of plane (seen from the $3^{rd.}$ quadrant).

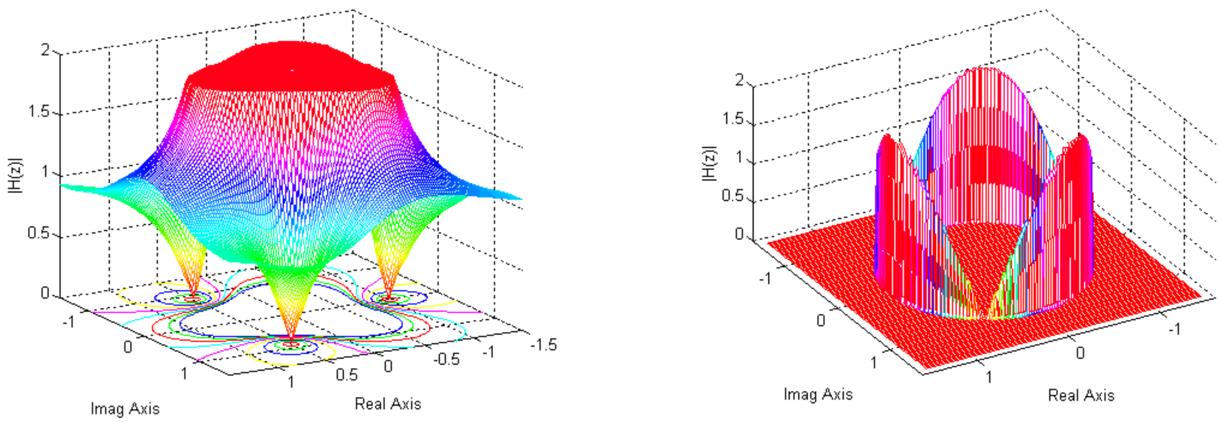


(a)



(b)

Figure 6. Absolute value of the transfer function of a second-order lowpass filter represented (a) over the *s* plane and (b) on the left-hand side of plane (seen from the 3rd. quadrant).



(a) (b)

Figure 7. Absolute value of the transfer function of a third-order digital "comb" filter represented (a) over the *z* plane and (b) on the unit circle (seen from the 1st. quadrant).

Improvements at CEFET-PR

CEFET-PR is an educational institution that offers different courses at the high-school, undergraduate and graduate levels. It offers the courses of Electrical Engineering, with emphasis on Electronics and Eletrotechnics, and Mechanical Engineering at the undergraduate level and, among others, Masters degrees in Informatics, Biomedical Engineering and Telematics at the graduate level. A Doctors level degree will be offered in these areas in a near future.

Many students were failing in disciplines like Control Systems and Communication Systems in previous years, when the basis of transforms was taught at the beginning of the semester, just following the sequence of most textbooks. Another discipline called Signals and Systems, in which special attention is given to the transforms and their applications, was then introduced. For economic reasons, the library does not have too many titles concerned with these subjects and the number of PCs that CEFET-PR has for teaching purposes is still limited. Hundreds of students don't have access to well known software because it is very expensive and is bought only for research purposes thanks to special financial aid offered by research councils.

Keeping in mind the idea that transfer functions should be represented both in 2D and 3D, some demonstrations have been given with software. A booklet containing a complete analysis of different types of transfer functions was printed and offered to undergraduate and graduate students. Figures 5, 6 and 7 are examples of illustrations included in the booklet and are related to a second-order lowpass filter, a second-order bandpass filter and a digital "comb" filter represented by equations 8, 9 and 10 respectively. The variation of amplitude is caused by the position of poles and zeros. In figures 5-(b) and 6-(b), the frequency responses that can be obtained with the Fourier Transform, are seen over the imaginary axis (j ω), while in figure 7-(b), it is represented over the unit circle.

$$H(s) = \frac{1}{s^2 + s + 1} \quad (8)$$

$$H(s) = \frac{s}{s^2 + s + 1} \quad (9)$$

$$H(z) = 1 + z^{-3} \quad (10)$$

Although there is no special statistical data available at present, some results are shown in table 1 for the discipline Signals and Systems, at the Engineering level, before and after using 3D graphics in classes. The students achieved to obtain higher marks and the number of passed students reached 100% twice. Only students that came regularly to classes were taken into account.

Teachers have also noticed a reasonable improvement in the way in which students manipulate calculations and interpret the obtained results in subsequent disciplines.

A low-cost software package is now under development to give the students a chance of doing exercises wherever he/she has access to a PC.

Table 1. Comparison of marks and percentage of passed students for the discipline Signals and Systems before and after using 3D graphics for representing transfer functions.

	USING 2D GRAPHICS	USING BOTH 2D AND 3D GRAPHICS					
YEAR-SEMESTER	1995-1-A	1995-1-B	1995-2-A	1996-1-A	1996-1-B	1996-2-A	1997-1-A
NUMBER OF STUDENTS	34	31	37	29	34	32	34
AVERAGE MARK	5,6	5,1	6,6	5,9	6,8	7,5	5,2
STANDARD DEVIATION	1,3	1,4	1,1	1,4	1,3	0,9	1,7
PASSED STUDENTS (%)	79,4	74,2	97,3	89,7	100,0	100,0	76,5

Conclusion

The representation of complex functions in three dimensions is a necessity, particularly in textbooks since they are easy to manipulate, don't need to be connected to a power supply or battery and can be taken everywhere by the reader.

When the students have never worked with 3D projections and the textbook doesn't have at least one, he/she will have to spend some time checking other references or trying to obtain a complex function by himself through a personal computer with dedicated software. Such a solution seems to be obvious and easy, but not in second or third-world countries, where personal computers and associated programs are very expensive for educational institutions and almost unobtainable for students, mainly because they have to be imported.

Even technical books have to be imported and the final price is almost three times the normal dollar price. In Brazil, there is a further problem in that very few technical books are written in, or translated to, Portuguese [2, 10, 14]. Unfortunately, some publishers have avoided publishing new technical textbooks that are not related to popular subjects such as Informatics. It has been much easier to import those publications concerned with Communication Systems, Control Systems, Image and Signal Processing, for example, through the Internet.

No matter how difficult it has been to teach technical subjects concerned with complex variables at CEFET-PR, results have demonstrated that the students must have the chance of interpreting equations with the help of several graphical representations in both 2D and 3D. This way, calculations that were being made in a mechanical way will then be made with more pleasure and satisfaction, showing clearly the relation between theory and practice, without a high degree of abstraction. Ultimately this leads to a higher level of understanding on the part of the students.

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